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STRUCTURAL OPTIMIZATION OF CORRUGATED CORE
AND WEB CORE SANDWICH PANELS SUBJECTED
TO UNIAXIAL COMPRESSION

by

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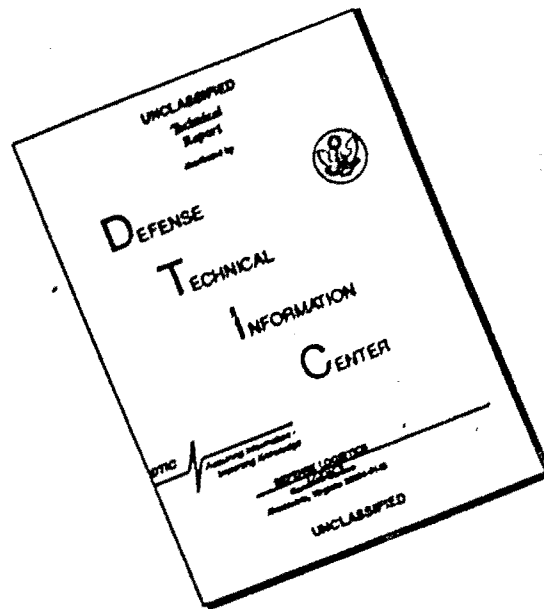
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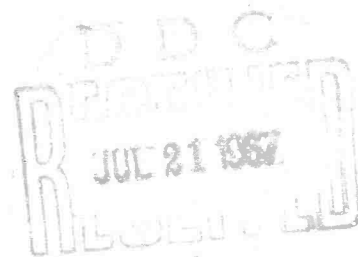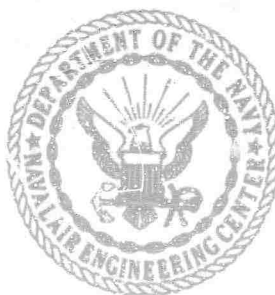
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FOREWORD

This report is one of four reports to be prepared by Structural Mechanics Associates under Navy Contract No. N156-46654. This contract was initiated under Work Unit No. 53C/07, "Development of Optimization Methods for the Design of Composite Structures Made from Anisotropic Material" (1-23-96) and was administered under the direction of the Aeronautical Structures Laboratory, Naval Air Engineering Center, with Messrs. R. Molella and A. Manno acting as Project Engineers. The reports resulting from this contract will be forwarded separately. Three reports are completed and cover work from 24 May 1965 to 31 December 1966. The title and approximate forwarding date for each report are as follows:

NAEC-ASL-1109, "Structural Optimization of Corrugated Core and Web Core Sandwich Panels Subjected to Uniaxial Compression," dated 15 May 1967. Forwarding date, June 1967.

NAEC-ASL-1110, "Structural Optimization of Flat, Corrugated Core and Web Core Sandwich Panels Under In-Plane Shear Loads and Combined Uniaxial Compression and In-Plane Shear Loads," dated 1 June 1967. Forwarding date, July 1967.

NAEC-ASL-1111, "A Method for Weight Optimization of Flat Truss Core Sandwich Panels Under Lateral Loads," dated 15 June 1967. Forwarding date, July 1967.

SUMMARY

This report presents the development of rational methods of structural optimization for flat, corrugated core and web-core sandwich panels subjected to uniaxial compressive loads.

These methods provide a means by which such panels can be designed with absolute minimum weight, yet retaining structural integrity (i.e., structurally optimum) for a given load index, panel width, panel length, and face and core materials. Of equal importance is that these methods provide a means for rational material selection through the comparison of weights of optimum construction for several material systems as a function of load index. The methods account for both isotropic or orthotropic face and core materials and various boundary conditions. Three types of core are considered: triangulated core (single truss core) construction, web-core construction, and "hat-shaped" core construction.

Chapter 1 presents the methods of optimization for the triangulated core (truss-core) construction. Chapter 2 presents methods of optimization for web-core construction. Chapter 3 presents methods of optimization for the "hat-shaped" core construction. Chapter 4 presents examples for several material systems using these constructions, as well as providing comparisons between these core constructions and honeycomb core construction. Chapter 5 presents some conclusions drawn from this investigation.

The methods developed herein are applicable to panels at elevated or lowered temperatures, under steady state and nearly uniform temperatures. Only the stress-strain curve, or preferably the tangent modulus-stress curve, for each temperature under consideration is necessary.

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NOTATION

a   Panel dimension in the x direction, in.

$\overline{A}_c$ Area of the core per unit width of corrugation crosssection parallel to the yz plane, in. (see Equations 1.1, 21., and 3.1)

b   Panel dimension in the y direction, in.

d_f Dimension in the y direction given in Figures 4 and 5

D_{qi} Transverse shear stiffness, per unit width, of a beam cut from the panel in the i direction (i=x,y), lbs./in. (see Equations 1.4, 1.5, 2.4, 3.4, and 3.5)

D_i $\frac{1}{2} E_i t_f h_c^2$, lbs.-in. (i=x,y)

D_j Flexural stiffnesses associated with an orthotropic plate, lbs.-in. (j=1,2,3)

E   Modulus of elasticity, lbs./in.^{2}

E_c Modulus of elasticity of corrugated core sheet material, lbs./in.^{2}

E_f Modulus of elasticity of face sheet material, lbs./in.^{2}

E_o Definition given by Equation 1.66

E_T Tangent modulus, lbs./in.^{2}

E^* $(E_c/E_f)\Omega$

$\overline{E}$ Reduced modulus of elasticity, lbs./in.^{2}

G_c Shear modulus of corrugated core sheet material, lbs./in.^{2}

h_c Core depth, in.

I Extensional stiffness of the facing sheets, lbs./in.

$\overline{I}_c$ Moment of inertia of the core, per unit width of the corrugation crossection parallel to the yz plane, taken about the centroidal axis of the corrugation crossection, in.^{3} (see Equations 1.2, 2.2, and 3.2)

$\overline{I}_f$ Moment of inertia per unit width of the faces considered as membranes, with respect to the sandwich plate middle surface, in.^{3} (see Equation 1.8, 2.6, and 3.6)

K Buckling coefficient

m Number of half waves in the x direction

M_o Definition given by Equation 1.79

N_x Compressive in-plane load in the x direction per unit panel width, lbs./in.

P $d_f + h_c \tan \theta$

r_i Transverse shear flexibility ratio (see Equation 2.8) $(i = x, y)$

S Definition given by Equations 1.6 and 3.5

t_c Thickness of core web, in.

t_f Thickness of facing material, in.

V Definition given by Equation 1.10 $(= r_y)$

W Total weight per unit planform area of panel construction, lbs./in.^{2}

W_i Weight per unit planform area of core (i=c) or facing (i=f) materials

W_{ad} Weight of adhesive or other joining material between facing and core per unit planform area, lbs./in.^{2}

x Panel in-plane coordinate (see Figure 2)

y Panel in-plane coordinate (see Figure 2)

z Panel coordinate normal to mid-plane of panel (see Figures 1 and 4)

β a/b

δ_i In-plane axial compressive deformation, in. (see Equation 1.16) i=c,f

ε In-plane strain

η Plasticity reduction factor

θ Angle web material makes with a line normal to plane of faces

ν Poisson's ratio

ρ_i Density, lbs./in.^{3} (i=c,f)

σ Stress, psi

Ω $\left[(1-\nu_c^2)/(1-\nu_F^2)\right]^{1/2}$

Ω_0 Definition given by Equation 1.80

CHAPTER 1

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT
TRIANGULATED CORE (SINGLE TRUSS CORE) SANDWICH
PANELS SUBJECTED TO UNIAXIAL COMPRESSION

A. Introduction

Consider a flat corrugated core sandwich panel involving the construction shown in crossection in Figure 1.
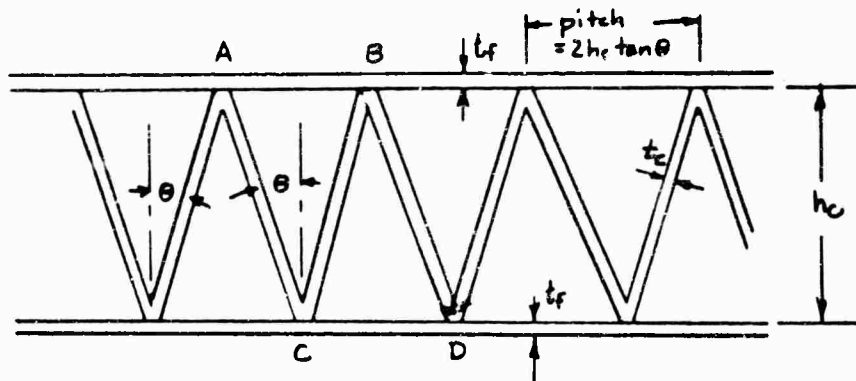
Figure 1

Triangulated Core Sandwich Panel

There are four geometric parameters with which to optimize; namely, the core depth (h_c), the web thickness (t_c), the face thickness (t_f), and the angle the web makes with a line normal to the faces (θ).

The overall panel to be considered is shown in planform in Figure 2.
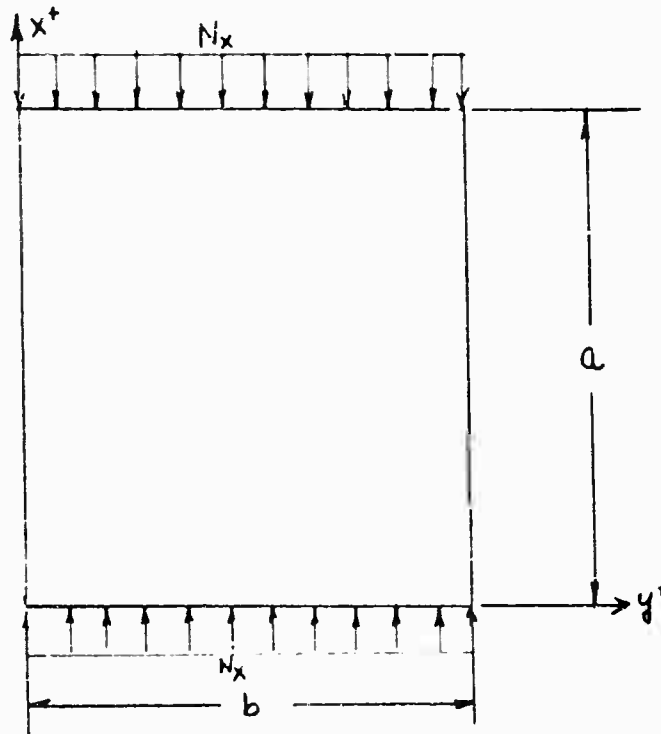
Figure 2

Planform View of Panel

This panel of width b and length a is subjected to an in-plane compressive load N_x (lbs./in.) parallel to the x axis. This panel is considered to fail if any of the following instabilities occur: overall instability, local face buckling, and web buckling. It can be shown that face wrinkling, in the sense of face wrinkling in honeycomb sandwich construction will not occur, because local face buckling and web buckling will invariably occur at lower values of the load. It can also be shown that core shear instability, in the sense of shear crimping

in honeycomb sandwich construction cannot occur, because a geometry would be required which would violate the construction considered, namely a pitch in excess of the panel width. Hence, there are three modes of instability and four geometric parameters.

To describe the instability, the analytical expression used in each case below is the best available from the literature.

B. **Elastic and Geometric Constants Associated with Triangulated
Core Construction** ,

The elastic and geometric constants for the triangulated
core construction can be determined from those given in more
general form by Libove and Hubka in Reference 1. For the core
construction given in Figure 1, the following constants are
obtained.

The area of the core per unit width of corrugation
crossection parallel to the yz plane, $\overline{A}_c$, is given by

$$\overline{A}_c = \frac{t_c}{\sin\theta} \quad , \quad (\text{in.}) \tag{1.1}$$

where the symbols are defined in Figure 1.

The moment of inertia of the core per unit width of
corrugation crossection parallel to the yz plane, taken about
the centroidal axis of the corrugation crossection, $\overline{I}_c$, is
seen to be,

$$\overline{I}_c = \frac{t_c h_c^2}{12 \sin\theta} = \overline{A}_c \frac{h_c^2}{12} \quad . \quad (\text{in.}^3) \tag{1.2}$$

The extensional stiffness of the plate in the x direction
EA_x, is given by,

$$EA_x = E_c \overline{A}_c + 2E_f t_f \quad (\text{lbs./in.}) \tag{1.3}$$

where E_c and E_f are the compressive moduli of elasticity of
the core and face material respectively.

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The transverse shear stiffness, per unit width in the y-direction of an element of the sandwich cut by two x-z planes, D_{q_x}, is found to be,

$$D_{q_x} = \frac{G_c t_c \cos\theta}{\tan\theta} \quad \text{(lb./in.)} \tag{1.4}$$

The transverse shear stiffness, per unit width in the x direction of an element of the sandwich cut by two y-z planes, D_{q_y}, is given by

$$D_{q_y} = \frac{8 h_c E_c}{(1-\gamma_c^2)} \frac{t_c^3}{h_c^3} \quad \text{(lb./in.)} \tag{1.5}$$

where γ_c is the Poisson's ratio of the core material.

$$8 = \frac{h_c^2}{t_c^2} \sin^2\theta \cos\theta \tag{1.6}$$

Hence substituting (1.6) into (1.5) results in

$$D_{q_y} = \frac{E_c t_c}{(1-\gamma_c^2)} \cos^2\theta \sin\theta \tag{1.7}$$

This expression agrees with that found by Anderson in Reference 2.

Lastly, the moment of inertia per unit width, $\overline{I}_f$, of the faces, considered as membranes with respect to the sandwich-plane middle surface, is seen to be

$$\overline{I}_f = \frac{t_f h_c^2}{2} \quad \text{(in.}^3) \tag{1.8}$$

where because $t_f \ll h_c$, the core depth h_c can be taken as the distance between the centerlines of the faces.

C. Governing Equations for Panels Composed of Isotropic Materials

1. Overall Instability

The best expression describing the overall instability of a corrugated core sandwich panel under uniaxial compressive loads is given by Seide in Reference 3. It is written as:

$$N_x = \frac{\pi^2 E_f \bar{I}_f}{b^2} K \qquad (1.9)$$

where K is the buckling coefficient derived and plotted in Reference 3.

This buckling coefficient is plotted in Figures 2 and 4 of Reference 3 as a function of the length to width ratio, a/b, for the cases of the unloaded edges simply-supported and clamped respectively, for various values of the transverse shear flexibility parameter V. This parameter is defined as:

$$V = \frac{\pi^2 E_f \bar{I}_f}{b^2 D_{Qy}} = \frac{\pi^2 E_f t_f (1-v_c^2) h_c^4}{2b^2 S E_c t_c^3}$$

$$= \frac{\pi^2 (1-v_c^2)}{2 \cos^2\theta \sin\theta} \left(\frac{t_f}{t_c}\right)\left(\frac{h_c}{b}\right)^2 \left(\frac{E_f}{E_c}\right) \qquad (1.10)$$

It is noted that the buckling coefficient for this construction has the same general characteristics as the corresponding coefficient of a homogeneous flat plate under the same loading; namely a minimum value occurs for the lowest mode in the neighborhood of a/b = 1. For longer plates, a/b > 1, successive minima occur for increasing numbers of half sine

waves in the axial direction, each minima having the same
value for the coefficient K. Hence just as in homogeneous plates,
this minimum value can be taken as the lower bound (conservative)
for all plates in which the length to width ratio is greater
than unity. Hence, for the K values used herein the lower bound
values are used as plotted in Figures 3 and 5 of Reference 3
for the unloaded edges simply-supported and clamped,
respectively. When utilizing Figures 3 and 5 of Reference 3,
a parameter $E_c\bar{I}_c / E_f\bar{I}_f$ also appears which can be easily
determined using Equations (1.2) and (1.8). For a panel with
isotropic materials, it can be written as

$$\frac{E_c\bar{I}_c}{E_f\bar{I}_f} = \frac{E_c}{E_f} \frac{t_c}{t_f} \frac{1}{6\sin\theta} \qquad (1.10a)$$

It should be remembered, however, that if a panel is
considered in which the length to width ratio is less than
unity, higher values of K can be used as given by Figures 2
and 4 of Reference 3.

An equation for overall instability is given in page 96
of ANC-23 (Reference 4), which differs from Equation (1.9) only
by a factor of $(1-\nu_f^2)$ in the denominator, where ν_f is the
Poisson's ratio of the face material. However, since the curves
used in ANC-23 are taken directly from Reference 3, it appears
logical to use the buckling equation from Reference 3, as given
in Equation (1.9).

2. Face Plate Instability

Looking at Figure 1, it is seen that each plate element of the face from A to B can buckle due to the axial load N_x. Since the support conditions at A and B, the unloaded edges are not known precisely, it is conservative to assume that they are simply supported. For such a case the lower bound of the buckling coefficient K for this condition, where the length to width ratio is greater than unity, is equal to 4. Anderson in Reference 5 discusses the effect of more complex buckling modes, due to interaction between face and core elements. However, Figure 5 of Reference 5 shows that at most the buckling coefficient would be 4.21 for simultaneous buckling of face and core elements. The well known buckling equation to describe this instability, written in terms of this construction, is:

$$\sigma_f = \frac{\pi^2 E_f}{12(1-\nu_f^2)} \frac{t_f^2}{h_c^2} \frac{1}{\tan^2\theta} \tag{1.11}$$

where σ_f is the stress in the face.

3. Web Plate Instability

Likewise the local plate elements of the core can become unstable due to the core being subjected directly to a portion of the axial loading, N_x. The conservative assumption is made here also that the web elements from A to C and B to C in Figure 1 are simply supported along the unloaded edges, since the actual boundary conditions are unknown and may vary

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due to care in fabrication and method of joining the web and faces. Hence, K = 4, for elements which have greater length in the x direction than the dimension from A to C.

In terms of the symbols of Figure 1, the plate buckling equation is

$$\sigma_c = \frac{\pi^2 E_c}{3(1-\nu_c^2)} \frac{t_c^2}{h_c^2} \cos^2\theta \tag{1.12}$$

where σ_c is the stress in the web element.

4. Load-Stress Relationship

For the construction given in Figure 1 it is seen that the load N_x is related to the stress in the face σ_f and the stress in the web σ_c by the following relationship.

$$N_x = \sigma_c \bar{A}_c + 2\sigma_f t_f \tag{1.13}$$

Further, it is assumed that the axial shortening of the core δ_c must equal the axial shortening of the faces δ_f due to the loading N_x. This in other terms means that the core and the faces undergo equal strains.

Let N_{x_f} be the load/in. carried by the faces
and N_{x_c} be the load/in. carried by the core
and N_x be the load/in. carried by the entire panel

Then $N_{x_c} + N_{x_f} = N_x \tag{1.14}$

and

$$N_{x_c} = \sigma_c \bar{A}_c \qquad , \quad N_{x_f} = 2t_f \sigma_f \tag{1.15}$$

The axial shortening of the core (δ_c) and the axial shortening of the faces (δ_f) are found to be

$$\delta_c = \frac{N_{x_c} \, a}{E_c \bar{A}_c} \quad , \quad \delta_f = \frac{N_{x_f} \, a}{2E_f t_f} \quad . \tag{1.16}$$

Since $\delta_c = \delta_f$, from (1.15) and (1.16) it is seen that the stress in the core is related to the face stress by the following:

$$\sigma_c = \sigma_f \frac{E_c}{E_f} \tag{1.17}$$

Thus using the expressions above the face stress is related to the load per inch by the expression

$$\sigma_f = \frac{N_x}{\left[\frac{E_c}{E_f} \frac{t_c}{\sin \theta} + 2t_f \right]} \tag{1.18}$$

Note that relationships (1.16), (1.17), and (1.18) are only valid when the stresses in the faces and core are below the proportional limit of both the face and core material. Above these values the following procedure is required.
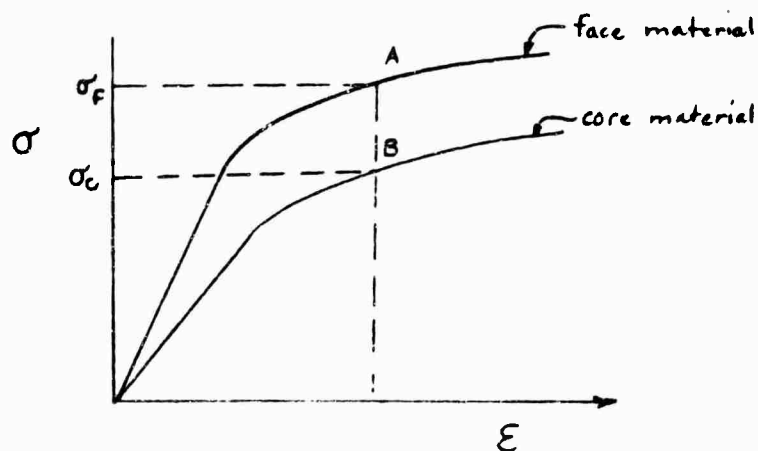
Figure 3

Stress Distribution in Face and Core
Above Proportional Limit

For any value of strain $\mathcal{E}$ corresponding to stress exceeding the proportional limit in either core or face, since the strains must be equal in both core and faces, a vertical line can be drawn on the stress-strain diagram as shown in Figure 3. The unique value of face stress and core stress can be determined for this strain.

Likewise in Figure 3, values of tangent moduli E_{f_t} and E_{c_t}, for the face and core respectively, can be determined, that are necessary to determine a plasticity reduction factor for use in the buckling expressions derived earlier.

5. Weight Relation

The weight relation is seen to be from Figure 1,

$$W = 2\rho_f t_f + \rho_c \bar{A}_c + W_{ad}$$

$$W - W_{ad} = 2\rho_f t_f + \frac{\rho_c t_c}{\sin \theta} \tag{1.19}$$

where ρ_f and ρ_c are the weight density of the face and core material respectively;

W_{ad} is the weight in lb/in^2 of planform a. v of the adhesive or other material used to join face and core;

W is the weight in lb/in^2 of planform area of the entire panel.

D. Structural Optimization of Panels with Faces and Core of the Same Isotropic Material

The governing equations pertaining to this construction are given by Equations (1.9), (1.11), (1.12) and (1.17), (1.18) and (1.19). In this case the simplifications in symbols are as follows: $E_c = E_f = E$; $\gamma_c = \gamma_f = \gamma$; $\sigma_c = \sigma_f = \sigma$. Because the stresses in the faces and core are equal when the same materials are used throughout, the methods derived below can be utilized for stresses above the proportional limit. $\bar{E}$ is used to indicate that a reduced modulus can be used. The results are:

$$N_x = \frac{\pi^2 \bar{E} t_f h_c^2 K}{2b^2} \tag{1.20}$$

$$\sigma = \frac{\pi^2 \bar{E} t_f^2}{12(1-\gamma^2)h_c^2 \tan^2\theta} \tag{1.21}$$

$$\sigma = \frac{\pi^2 \bar{E} t_c^2 \cos^2\theta}{3(1-\gamma^2) h_c^2} \tag{1.22}$$

$$N_x = \sigma \left[2t_f + \frac{t_c}{\sin\theta} \right] \tag{1.23}$$

$$W - W_{ad} = \rho \left[2t_f + \frac{t_c}{\sin\theta} \right] \tag{1.24}$$

The philosophy of optimization is as follows: a truly optimum structure is one which has a unique value for each variable within the class of structures being studied (triangulated core sandwich for example), for each set of materials (7075-T6 aluminum for example), which will result in the minimum possible weight for a specified set of loads (uniaxial compressive load per unit width, N_x for example), and yet maintain structural integrity (will have no mode of failure less than that occurring at the design load). In this case the optimum structure will have the characteristic that the panel will become unstable in all three buckling modes simultaneously. If this is not the case then one of the critical modes corresponding to a face stress, say σ_1 , occurs at a higher value of face stress than the other two, say $\sigma_2 = \sigma_3 < \sigma_1$ However, the panel will fail at a load corresponding to the lower face stresses σ_2 and σ_3, say $N_2 = N_3$. This in turn means that there exists material (which has weight) in the structure which is not being stressed or strained sufficiently for it to be used most efficiently. Thus there are two alternatives available. (1) Material can be removed until the failure mode originally occurring at σ_1 is reduced to the stress $\sigma_2 = \sigma_3$, in which case a lighter structure results for an applied load $N_2 = N_3$. (2) Material can be rearranged reducing the critical stress value originally at σ_1 corresponding to the first mode to some value σ_4 , while raising the critical face stress values associated with the other two failure modes originally occurring at σ_2 and σ_3

to a value σ_4 ($\sigma_1 > \sigma_4 > \sigma_2 = \sigma_3$). Now the structure with the same weight as the original arrangement can withstand a load N_4 (corresponding to σ_4) where $N_4 > N_2 = N_3$. Obviously both (1) and (2) can be performed simultaneously so that some material is removed and some rearranged, resulting in an optimum structure.

Returning to the Equations (1.20) through (1.24), the known specified quantities are the applied load per inch N_x and the panel width b, which can be lumped together as the load index N_x/b; the material properties E, γ, and ρ. The buckling coefficient K is a slowly varying function of the dependent variables that for the present will be considered as a constant, but will be discussed later.

The dependent variables in the set with which to optimize the construction are the face thickness, t_f; the core depth, h_c; the web thickness, t_c; the web angle, θ; the stress, σ; and the weight, $W - W_{ad}$.

It is seen that there are five equations and six unknowns. The sixth equation is obtained by placing the weight equation in terms of one convenient variable, taking the derivative of the weight equation with respect to this variable, and setting it equal to zero to obtain the unique value of the variable resulting in minimum weight.

It should be noted that this last step differs from the optimization procedure given by Vinson and Shore in Reference 6 for honeycomb sandwich panels under the same loading.

In that construction there are six governing equations and
six dependent variables. Thus, employing the optimization
philosophy expressed in Reference 6 and repeated earlier in
this section, the optimization weight is directly determined
by equating the stresses for each failure mode.

Before proceeding it should be noted also that in
this case where the core and face material are identical, the
optimization procedure can be carried out directly for stresses
above the proportional limit of the material in the same way
as in the range where Hooke's Law applies. It is only necessary
to introduce a suitable plasticity reduction factor, η, such
that

$$\overline{E} = \eta E \tag{1.25}$$

In Equations 1.20 through 1.24 as well as in the following
$\overline{E}$ will replace E for generality.

Manipulation of Equations (1.20) through (1.24)
results in an expression for the weight equation in terms of
the web angle θ, as shown below:

$$\frac{W - W_{ad}}{b} = \left[\frac{3(1 - v^2)}{K}\right]^{1/4} \left[\frac{N_x/b}{\overline{E}}\right]^{1/2} \frac{\rho}{\pi} \frac{\left[1 + 4 \sin^2\theta\right]^{3/4}}{\sin\theta (\cos\theta)^{1/2}} . \tag{1.26}$$

As stated previously, the sixth equation to obtain
minimum weight consists of the derivative of the weight
equation with respect to the single variable, equated to zero.

Taking the derivative of (1.26) with respect to θ and equating it to zero results in the important relationship

$$\sin^2\theta = 2/7$$

Therefore, $\theta \approx 32.40^\circ$

$$(1.27)$$

It is seen that to obtain minimum weight structure for a triangulated core panel involving faces and cores of the same isotropic materials, the web angle is constant and independent of isotropic materials used, the coefficient K, and and the magnitude of the Load Index (N_x/b).

One may now obtain the "universal relationship" relating load index to a unique value of stress, for any set of material properties, which will result in minimum weight. For this construction it is seen that the universal relationship using equations 1.20 through 1.24 is

$$\frac{N_x}{b} = \frac{45}{2} \frac{(1-\nu^2)^{1/2} \sigma^2}{\pi^2 \bar{E} K^{1/2}}$$

$$(1.28)$$

The geometric variables t_f, t_c, and h_c as well as the weight equation can now be expressed in terms of the optimum stress σ, or the load index (N_x/b). Simpler expressions are obtained when the optimum stress is used, which of course is related to the load index through equation (1.28). These expressions are:

$$\frac{t_f}{b} = \frac{6(1-\nu^2)^{1/2} \sigma}{\pi \bar{E} K^{1/2}}$$

$$(1.29)$$

$$\frac{t_{c}}{b} = \frac{3\sqrt{\frac{7}{2}} \left(1-\nu^{2}\right)^{1/2} \sigma}{\pi^{2} E K^{1/2}} \tag{1.30}$$

$$\frac{h_{c}}{b} = \left[\frac{15}{2} \frac{\sigma}{\pi^{2} \bar{E} K}\right]^{1/2} \tag{1.31}$$

$$\frac{W - W_{ad}}{b} = \frac{45}{2} \frac{\left(1-\nu^{2}\right)^{1/2} \rho \sigma}{\pi^{2} \bar{E} K^{1/2}} \tag{1.32}$$

In terms of the given load index (N_x/b) the expressions are seen to be:

$$\frac{t_{F}}{b} = \frac{2\sqrt{2}}{\sqrt{5}} \frac{\left(1-\nu^{2}\right)^{1/4} \left(N_x/b\right)^{1/2}}{\pi \bar{E}^{1/2} K^{1/4}} \tag{1.33}$$

$$\frac{t_{c}}{b} = \sqrt{\frac{7}{5}} \frac{\left(1-\nu^{2}\right)^{1/4} \left(N_x/b\right)^{1/2}}{\pi \bar{E}^{1/2} K^{1/4}} \tag{1.34}$$

$$\frac{h_{c}}{b} = \left[\frac{5 \left(N_x/b\right)}{2 \bar{E} \pi^{2} \left(1-\nu^{2}\right)^{1/2} K^{3/2}}\right]^{1/4} \tag{1.35}$$

$$\frac{W - W_{ad}}{b} = 3 \left(\frac{5}{2}\right)^{1/2} \frac{\left(1-\nu^{2}\right)^{1/4} \rho \left(N_x/b\right)^{1/2}}{\pi \bar{E}^{1/2} K^{1/4}} \tag{1.36}$$

Other interesting relationships are noticed for this construction where optimized. From (1.29) and (1.30) or (1.33) and (1.34), the ratio of the web thickness of the core to the face thickness is

$$t_{c}/t_{f} = \sqrt{7/8} \tag{1.37}$$

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It is also seen from (1.23) and (1.24) and by (1.28) and (1.32) that

$$\frac{N_x}{b} = \left(\frac{W - W_{ad}}{b}\right)\frac{\sigma}{\rho} \quad ; \quad \frac{W - W_{ad}}{b} = \left(\frac{N_x}{b}\right)\frac{\rho}{\sigma} \tag{1.38}$$

Looking at Equation (1.24), it is seen that for this construction the ratio of the weight of the core, W_c, to the weight of the facings, W_f, can be easily computed for optimum construction, utilizing (1.27) and (1.37):

$$\frac{W_c}{W_f} = \frac{t_c}{2t_f \sin\theta} = \frac{7}{8} \tag{1.39}$$

All quantities are now known, except for the precise determination of the constant K which, as mentioned earlier, is a slowly varying function of the dependent variables, but only appears in the weight equation to the $\frac{1}{4}$ power. In Figure 3 or 5 of Reference 3, values of K are tabulated as functions of the quantities $E_c\bar{I}_c/E_f\bar{I}_f$ and $V = \pi^2 E_f \bar{I}_f / b^2 D_{q\gamma}$, the core transverse shear flexibility parameter both of which must be determined. From Equations (1.2), (1.7), (1.8), (1.29), (1.30) and (1.31) these quantities become:

$$\frac{\bar{E}_c \bar{I}_c}{\bar{E}_f \bar{I}_f} = \frac{7}{24} \tag{1.40}$$

$$V = \frac{21(1-v^2)\sigma}{2\bar{E}K} \tag{1.41}$$

By a simple iteration involving assumption of K, solving for V for a given set of materials and stress, reading K from the Figure, and repeating, the value of K can be determined quite rapidly. It is seen immediately from (1.41) that the core flexibility parameter is very low due to the ratio $\sigma/\bar{E}$, hence K will always be very near the value given by the intercept with the ordinate on the aforementioned figures.

The design procedures will be given in summary form in Section I of this chapter.

E. Structural Optimization of Panels with Faces and Core of Different Isotropic Materials

The governing equations pertaining to this construction are given by Equations (1.9), (1.11), (1.12), (1.17), (1.18), and (1.19), and are listed below in summary.

$$N_x = \frac{\pi^2 E_f \, t_f \, h_c^2 \, K}{2b^2} \tag{1.42}$$

$$\sigma_f = \frac{\pi^2 E_f \, t_f^2}{12(1 - \nu_f^2) h_c^2 \, \tan^2 \theta} \tag{1.43}$$

$$\sigma_f = \frac{\pi^2 E_f \, t_c^2 \, \cos \theta}{3(1 - \nu_c^2) h_c^2} \tag{1.44}$$

$$N_x = \sigma_f \left[\frac{E_c}{E_f} \frac{t_c}{\sin \theta} + 2t_f \right] \tag{1.45}$$

$$W - W_{ad} = 2 \rho_f t_f + \frac{\rho_c t_c}{\sin \theta} \tag{1.46}$$

The philosophy of the optimization is identical to that expressed in Section D of this chapter, i.e., all independent modes of failure that cause the panel to be considered structurally unsound, must occur simultaneously to produce a minimum weight structure.

Again it is seen that there are six unknown quantities (t_f, t_c, h_c, θ, σ_f, and $W - W_{ad}$) and five equations. The sixth

equation is obtained by expressing the weight equation in terms
of a convenient variable, and setting the derivative of the
weight equation (1.46) with respect to this variable equal to
zero to obtain the minimum weight value for that variable.

In the case in which face and core materials differ,
the expressions given by Equations (1.17), (1.18), and (1.44)
are applicable where both the face and core stresses are below
the proportional limit. Above the proportional limit, an
iteration procedure involving the equating of strains as shown
in Figure 3, and using the form of the web buckling equation
shown in (1.12) must be employed. This shall be further
discussed in Section H dealing with optimization procedures.

The remainder of this section deals with the explicit
solution of the problem for stresses below the proportional
limit of both materials.

Manipulations of Equations (1.42) through (1.46)
can result in the weight equation being expressed entirely in
terms of known quantities and the unknown web angle Θ below,

$$\frac{W - W_{ad}}{b} = \frac{3^{1/4} \left(1 - \nu_F^2\right)^{1/4} \rho_F \left(\frac{N_x}{b}\right)^{1/2}}{\pi E_f^{1/2} K^{1/4}} \left\{ \frac{\left(\rho^* + 4 \sin^2\Theta\right)}{\sin\Theta \left(\cos\Theta\right)^{1/2} \left(E^* + 4 \sin^2\Theta\right)^{1/4}} \right\} \qquad (1.47)$$

where

$$\rho^* = \frac{\rho_c}{\rho_F} \, \Omega$$

$$E^* = \frac{E_c}{E_f} \, \Omega$$

and

$$\Omega = \left[\frac{1 - \nu_c^2}{1 - \nu_f^2}\right]^{1/2}$$

As stated previously, the sixth equation is developed

by setting the derivative of Equation (1.47) with respect to
θ equal to zero. The result is

$$\left[8 + 8\rho^* - 2E^*\right] \sin^4\theta - \left[6\rho^* - 4E^* - \frac{3}{2}\rho^* E^*\right] \sin^2\theta$$
$$- \rho^* E^* = 0 \qquad (1.48)$$

For a given material system solving (1.48) for θ gives
the value of θ which will insure an optimum construction. Note
that this value of θ is independent of the load index (N_x/b),
and the buckling coefficient K, which in turn means it is
independent of the panel boundary conditions on the unloaded
edges (simply supported or clamped).

The "universal relationship" can now be found,
knowing θ, which gives the unique value of face stress assoc-
iated with minimum weight for a given load index and set of
material properties.

$$\frac{N_x}{b} = \left[\frac{3(1-\nu_F^2)}{K}\right]^{1/2} \frac{\sigma_F^2}{\pi^2 E_F} \frac{\left[E^* + 4\sin^2\theta\right]^{1/2}}{\sin^2\theta \cos\theta} \qquad (1.49)$$

The geometric variables t_f, t_c, and h_c as well as
the weight equation can now be expressed in terms of the
optimum stress σ_F, or the load index (N_x/b). The simple
expressions result in those expressions in which the optimum
face stress is used, as given below.

$$\frac{t_f}{b} = \frac{2\sqrt{3}(1-\nu_F^2)^{1/2}\sigma_F}{\pi^2 E_F K^{1/2} \cos\theta} \left[E^* + 4\sin^2\theta\right]^{1/2} \qquad (1.50)$$

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$$\frac{t_c}{b} = \frac{\sqrt{3}\left(1-v_c^2\right)^{1/2}\sigma_F}{\pi^2 E_f K^{1/2}} \frac{\left[E^* + 4\sin^2\theta\right]^{1/2}}{\sin\theta\,\cos\theta} \tag{1.51}$$

$$\frac{h_c}{b} = \frac{\sigma_F^{1/2}}{\pi E_f^{1/2} K^{1/2}} \frac{\left[E^* + 4\sin^2\theta\right]^{1/2}}{\sin\theta} \tag{1.52}$$

$$\frac{W-W_{ad}}{b} = \frac{\sqrt{3}\left(1-v_F^2\right)^{1/2}\rho_F\,\sigma_F}{\pi^2 E_f K^{1/2}} \frac{\left[\rho^* + 4\sin^2\theta\right]\left[E^* + 4\sin^2\theta\right]^{1/2}}{\sin^2\theta\,\cos\theta} \tag{1.53}$$

The geometric variables in terms of the specified load index are given below. The weight equation in terms of the load index is given by (1.47).

$$\frac{t_F}{b} = \frac{2(3)^{1/4}\left(1-v_F^2\right)^{1/4}\left(N_x/b\right)^{1/2}}{\pi E_f^{1/2} K^{1/4}} \frac{\sin\theta}{(\cos\theta)^{1/2}\left[E^*+4\sin^2\theta\right]^{1/4}} \tag{1.54}$$

$$\frac{t_c}{b} = \frac{(3)^{1/4}\,\Omega\,\left(1-v_F^2\right)^{1/4}\left(\frac{N_x}{b}\right)^{1/2}}{\pi E_f^{1/2} K^{1/4}(\cos\theta)^{1/2}\left[E^*+4\sin^2\theta\right]^{1/4}} \tag{1.55}$$

$$\frac{h_c}{b} = \frac{\left(N_x/b\right)^{1/4}}{(3)^{1/8}\pi^{1/2}\left(1-v_F^2\right)^{1/8}E_f^{1/4}K^{3/8}} \cdot \frac{(\cos\theta)^{1/4}\left[E^*+4\sin^2\theta\right]^{1/8}}{(\sin\theta)^{1/2}} \tag{1.56}$$

One useful relationship is the relationship between the web thickness to the facing thickness, which is easily seen to be

$$\left(\frac{t_c}{b}\right) = \frac{\Omega\left(t_F/b\right)}{2\sin\theta} \tag{1.57}$$

As before it is now necessary to determine the precise value of the buckling coefficient K, from Figures 3 or 5 of Reference 3. It is only necessary to know first the ratios $E_c \bar{I}_c / E_f \bar{I}_f$ and $V = \pi^2 E_f \bar{I}_c / b^2 D_{qf}$ for the optimized construction. From the foregoing these values are found to be:

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{E_c}{E_f} \frac{\Omega}{12 \sin^2 \theta} \tag{1.58}$$

and

$$V = \frac{(1-\nu_c^2) \, \sigma_f \left[E^* + 4 \sin^2 \theta \right]}{E_c \, \Omega K \, \sin^2 \theta \, \cos^2 \theta} . \tag{1.59}$$

A very rapidly converging iteration results when these equations are used with Figures 3 or 5 of Reference 3 to determine K. In optimum construction V , the transverse shear core flexibility parameter will usually be very small.

The actual design procedures will be given in summary form in Section I of this chapter.

F. Governing Equations for Panels Composed of Orthotropic Materials

1. Overall Instability

As of the date of this report there is no published study of the overall stability of corrugated core sandwich panels employing orthotropic materials under uniaxial compressive loads. However, it is not difficult to deduce the form of the expression for overall instability by reference to closely related analyses.

On page 53 of ANC-23 (Reference 4) the expression for overall instability of a honeycomb sandwich panel utilizing isotropic materials under uniaxial compression is given by

$$\sigma_f = \frac{E_f \pi^2 D K}{b^2 H} \qquad (1.60)$$
$$\text{where } H = 2 E_f t_F.$$

On page 96 of ANC-23 the expression for the overall instability of a corrugated core sandwich panel utilizing isotropic materials under uniaxial compression is given by

$$\sigma_f = \frac{E_f \pi^2 D K}{b^2 H} \qquad (1.61)$$

where $H = E_{f_x}\left[2 t_F + (t_c/\sin\theta)\right]$ for the corrugated core sandwich construction. The expressions seem to be identical in form.

On page 82 of ANC-23, the expression for the overall instability of a honeycomb core sandwich panel utilizing orthotropic materials under uniaxial compression is given by

$$\sigma_f = E_{fx} \frac{\pi^2 \sqrt{D_x D_y}}{b^2 H_x} \tag{1.62}$$

where $H_x = 2E_{fx}t_f$

Comparing this with Equation (1.60) it is seen that concerning flexural properties E_f is replaced by $\sqrt{E_{fx}E_{fy}}$ while in extensional properties E_f is replaced by E_{fx}.

Therefore, from the foregoing it is hypothesized that for a corrugated core sandwich panel utilizing orthotropic materials the equation will be identical in form to (1.62), namely $\sigma_f = E_{fx} \frac{\pi^2 \sqrt{D_x D_y}}{b^2 H_x}$,
or in terms of the load/inch, N_x will be

$$N_x = \frac{\pi^2 \sqrt{E_{fx} E_{fy}} \, t_f h_c^2 K}{2 b^2} . \tag{1.63}$$

Here, $N_x = \sigma_f H_x$, and in being consistent with the previous discussion in Section C, concerning $D = E_f \overline{I}_f$, then

$$\sqrt{D_x D_y} = \sqrt{E_{fx}E_{fy}} \ \overline{I}_f = \sqrt{E_{fx}E_{fy}} \ t_f h_c^2/2 .$$

Analogous to Section C-1, it is also hypothesized that the curves 3 and 5 of Reference 3 may also be used to determine the value of the buckling coefficient K by suitable transformation of the isotropic material properties, to those of orthotropic materials. The replacements correspond to

those made to obtain Equation (1.63). Hence

$$V = \frac{\pi^2 \left(1 - \nu_{xyc}\, \gamma_{yxc}\right)}{2 \cos^2\theta \, \sin\theta} \left(\frac{t_F}{t_c}\right)\left(\frac{h_c}{b}\right)^2 \frac{\sqrt{E_{F_x} E_{F_y}}}{\sqrt{E_{cx} E_{cy}}} \qquad (1.64)$$

$$\frac{E_c \bar{I}_c}{E_F \bar{I}_F} = \frac{\sqrt{E_{cx} E_{cy}}}{\sqrt{E_{F_x} E_{F_y}}} \; \frac{t_c}{t_F} \; \frac{1}{6 \sin\theta} \qquad (1.65)$$

2. Face Plate Instability

Timoshenko and Gere (Reference 7, p. 404) present an expression for the critical stress for an orthotropic plate, simply supported on all edges, of width b and length a (a b), and thickness h, subjected to a uniaxial load in the lengthwise direction.

The expression given is

$$\sigma = \frac{2\pi^2}{b^2 h} \left(\sqrt{D_1 D_2} + D_3\right)$$

where

$$D_1 = \frac{E_x h^3}{12\left(1 - \nu_{xy}\nu_{yx}\right)}$$

$$D_2 = \frac{E_y h^3}{12\left(1 - \nu_{xy}\nu_{yx}\right)}$$

$$D_3 = \frac{1}{2}\left(\nu_{xy} D_2 + \nu_{yx} D_1\right) + 2\left(GI\right)_{xy}$$

Defining a constant E_0 such that

$$2E_0 = \sqrt{E_x E_y} + \nu_{yx} E_x + 2G_{xy}\left(1 - \nu_{xy}\nu_{yx}\right) \qquad (1.66)$$

the critical stress can be presented in the familiar form for a

simply supported plate under uniaxial compression in the length-
wise direction, namely,

$$\sigma = \frac{\pi^2 E_o h^2}{3(1-\nu_{xy}\nu_{yx})b^2} \, . \tag{1.67}$$

For the local facing instability of the triangulated
core construction, from Figure 1, the expression is easily
determined to be,

$$\sigma_f = \frac{\pi^2 E_{of} t_f^2}{12 h_c^2 (1-\nu_{xyf}\nu_{yxf}) \tan^2\theta} \, , \tag{1.68}$$

where the subscripts f refer to the facing material.

3. Web Plate Instability

Again from the geometry of Figure 1, it is seen that
for the web plates Equation (1.67) can be written as

$$\sigma_c = \frac{\pi^2 E_{oc} t_c^2 \cos^2\theta}{3 h_c^2 (1-\nu_{xyc}\nu_{yxc})} \, , \tag{1.69}$$

where the subscript c refers to the core material.

4. Load-Stress Relationship

Analogous to Equations (1.17) and (1.18), replacing
the E values by E_x values in _extensional_ quantities, the
corresponding relationships for orthotropic materials are

$$\sigma_c = \sigma_f \frac{E_{cx}}{E_{fx}} \tag{1.70}$$

and

$$N_x = \sigma_f \left[\frac{E_{cx}}{E_{fx}} \frac{t_c}{\sin\theta} + 2 t_f \right] . \tag{1.71}$$

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Equation (1.71) is consistent with Equation A, page 80 of ANC-23 (Reference 4).

Again, it should be noted what when the face material differs from that of the core material, the expressions (1.70) and (1.71) pertain only to the range of loads and stresses less than the proportional limit of both the face and core material. When either face or core materials are stressed beyond the proportional limit an iterative procedure must be employed utilizing the properties described in Figure 3.

5. Weight-Relation

As before the weight equation for this type of construction is given by

$$W \cdot W_{ad} = 2\rho_F t_F + \frac{\rho_c t_c}{\sin \theta} .$$

(1.72)

G. Structural Optimization of Panels in Which Faces and Core are Composed of Different Orthotropic Materials

For brevity, the structural optimization for ortho-tropic panels will be performed first for the case in which the face and core materials differ. The governing equations with which to optimize are given by Equations (1.63), (1.68), (1.69), (1.70), (1.71), and (1.72), and summarized below:

$$N_x = \frac{\pi^2 \sqrt{E_{Fx} E_{Fy}}\ t_F h_c^2 K}{2b^2} \tag{1.73}$$

$$\sigma_F = \frac{\pi^2 E_{of} t_F^2}{12(1-\nu_{xyF}\nu_{yxF}) h_c^2 \tan^2\theta} \tag{1.74}$$

$$\sigma_F = \frac{E_{Fx}}{E_{cx}} \frac{\pi^2 E_{oc}}{3} \frac{t_c^2}{h_c^2} \frac{\cos^2\theta}{(1-\nu_{xyc}\nu_{yxc})} \tag{1.75}$$

$$N_x = \sigma_F \left[\frac{E_{cx}}{E_{Fx}} \frac{t_c}{\sin\theta} + 2t_F \right] \tag{1.76}$$

$$W - W_{ad} = 2\rho_F t_F + \frac{\rho_c t_c}{\sin\theta} \tag{1.77}$$

The philosophy of optimization is identical to that expressed in Sections D and E of this chapter, namely, all independent modes of failure that cause the panels to be con-sidered structurally unsound must occur simultaneously to produce a panel of minimum weight.

Again there are six unknowns and five equations.
The sixth equation is obtained by expressing the weight equation
in terms of a convenient variable, and setting the derivative
of the weight equation with respect to this variable equal to
zero to obtain the value of that variable which will ensure
minimum weight.

Algebraic manipulations of Equations (1.73) through
(1.77) can produce the weight equation as a function of the
angle Θ only, as follows:

$$\frac{W-W_{ad}}{b} = \frac{(3)^{1/4} \rho_F \left(N_x/b\right)^{1/2} \left(1-\nu_{xyF} \nu_{yxF}\right)^{1/4}}{K^{1/4} \pi \left(E_{Fx} E_{Fy}\right)^{1/8} E_{0F}^{1/4} \sin \Theta \left(\cos \Theta\right)^{1/2}} \times$$

$$\frac{\left\{ 4 \sin^2 \Theta + \left(\frac{\rho_c}{\rho_F}\right) \left(\frac{E_{cx}}{E_{Fx}}\right)^{1/2} \left(\frac{E_{0F}}{E_{0c}}\right)^{1/2} \Omega_0 \right\}}{M_0^{1/4}}$$

(1.78)

$$\text{where} \quad M_0 = \left[\left(\frac{E_{cx}}{E_{Fx}}\right)^{3/2} \left(\frac{E_{0F}}{E_{0c}}\right)^{1/2} \Omega_0 + 4 \sin^2 \Theta\right]$$

(1.79)

$$\text{and} \quad \Omega_0 = \left[\frac{1-\nu_{xyc} \nu_{yxc}}{1-\nu_{xyF} \nu_{yxF}}\right]^{1/2}$$

(1.80)

Taking the derivative of Equation (1.78) with respect
to Θ and setting the result equal to zero, provides the equation
to determine Θ which will insure minimum weight. This equation
can be written as:

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$$\left\{ 8 + 8 \left(\frac{\rho_c}{\rho_f}\right) \left(\frac{E_{cx}}{E_{fx}}\right)^{1/2} \left(\frac{E_{of}}{E_{oc}}\right)^{1/2} \Omega_0 - 2 \left(\frac{E_{cy}}{E_{fy}}\right)^{3/2} \left(\frac{E_{of}}{E_{oc}}\right)^{1/2} \Omega_0 \right\} \sin^4 \theta$$

$$- \left\{ 6 \left(\frac{\rho_c}{\rho_f}\right) \left(\frac{E_{cx}}{E_{fx}}\right)^{1/2} \left(\frac{E_{of}}{E_{oc}}\right)^{1/2} \Omega_0 - 4 \left(\frac{E_{cx}}{E_{fx}}\right)^{3/2} \left(\frac{E_{of}}{E_{oc}}\right)^{1/2} \Omega_0 \right.$$

$$\left. - \frac{3}{2} \left(\frac{\rho_c}{\rho_f}\right) \left(\frac{E_{cx}}{E_{fx}}\right)^2 \left(\frac{E_{of}}{E_{oc}}\right) \Omega_0^2 \right\} \sin^2 \theta$$

$$- \left\{ \left(\frac{\rho_c}{\rho_f}\right) \left(\frac{E_{cx}}{E_{fx}}\right)^2 \left(\frac{E_{of}}{E_{oc}}\right) \Omega_0^2 \right\} = 0 \tag{1.81}$$

Note that, again, the value of θ producing a minimum weight panel is independent of the load index $(N_{x/b})$ and the buckling coefficient K. It is therefore also independent of the boundary conditions.

The "universal relationship" relating the unique value of face stress to a given applied load index $(N_{x/b})$ is given by

$$\frac{N_x}{b} = \frac{3^{1/2} M_0^{3/2} \sigma_f^2 \left(1 - \nu_{xyf} \nu_{yxf}\right)^{1/2}}{\pi^2 K^{1/2} E_{of}^{1/2} \left(E_{fx} E_{fy}\right)^{1/4} \sin^2 \theta \cos \theta} \tag{1.82}$$

where θ is known from (1.81).

The geometric variables t_f, t_c and h_c can now be expressed in terms of the optimum face stress determined by (1.82), or the load index $(N_{x/b})$. The simpler of the two sets is in terms of σ_f and θ and is given as follows:

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$$\frac{t_F}{b} = \frac{2\sqrt{3}\ \sigma_F\ M_0^{1/2}\left(1-\nu_{xyf}\ \nu_{yxf}\right)^{1/2}}{K^{1/2}\ \pi^2\ E_{of}^{1/2}\ \cos\Theta\ \left(E_{fx}\ E_{fy}\right)^{1/4}} \tag{1.83}$$

$$\frac{t_c}{b} = \frac{\sqrt{3}\left(\dfrac{E_{cx}}{E_{fx}}\right)^{1/2}\ \sigma_F\ \left(1-\nu_{xyc}\ \nu_{yxc}\right)^{1/2}\ M_0^{1/2}}{\pi^2\ E_{oc}^{1/2}\ K^{1/2}\ \left(E_{fx}\ E_{fy}\right)^{1/4}\ \sin\Theta\ \cos\Theta} \tag{1.84}$$

$$\frac{h_c}{b} = \frac{\sigma^{1/2}\ M_0^{1/2}}{\pi\ K^{1/2}\left(E_{fx}\ E_{fy}\right)^{1/4}\ \sin\Theta}. \tag{1.85}$$

The same variables expressed in   rms of the load index $(\bar{N}_{x/b})$ and Θ can be written as,

$$\frac{t_F}{b} = \frac{2\ (3)^{1/4}\ \sin\Theta\ \left(N_{x/b}\right)^{1/2}\left(1-\nu_{xyf}\ \nu_{yxf}\right)^{1/4}}{K^{1/4}\ \pi\ E_{of}^{1/4}\ \left(\cos\Theta\right)^{1/2}\left(E_{fx}\ E_{fy}\right)^{1/8}\ M_0^{1/4}} \tag{1.86}$$

$$\frac{t_c}{b} = \frac{(3)^{1/4}\left(\dfrac{E_{cx}}{E_{fx}}\right)^{1/2}\ E_{of}^{1/4}\left(N_{x/b}\right)^{1/2}\ \Omega_0^{1/2}\left(1-\nu_{xyc}\ \nu_{yxc}\right)^{1/4}}{\pi\ E_{oc}^{1/2}\left(E_{fx}\ E_{fy}\right)^{1/8}\ K^{1/4}\ M_0^{1/4}\left(\cos\Theta\right)^{1/2}} \tag{1.87}$$

$$\frac{h_c}{b} = \frac{E_{of}^{1/8}\left(\cos\Theta\right)^{1/4}\ M_0^{1/8}\left(N_{x/b}\right)^{1/4}}{3^{1/8}\ K^{3/8}\ \pi^{1/2}\left(E_{fx}\ E_{fy}\right)^{3/16}\left(\sin\Theta\right)^{1/2}\left(1-\nu_{xyf}\ \nu_{yxf}\right)^{1/8}} \tag{1.88}$$

One other useful relationship is found to be the core thickness as a function of face thickness. From (1.83) and (1.84) or (1.86) and (1.87),

$$\frac{t_c}{b} = \left(\frac{E_{oF}}{E_{oc}}\right)^{1/2} \left(\frac{E_{cx}}{E_{Fx}}\right)^{1/2} \frac{\Omega_o}{2\sin\theta} \left(\frac{t_F}{b}\right) \tag{1.89}$$

As before, it is necessary to determine the buckling coefficient K from Figures 3 or 5 of Reference 3. First we must determine the flexural stiffness ratio and the transverse shear core flexibility parameter. They can be found from the foregoing and are written as,

$$\frac{E_c I_c}{E_F I_F} = \frac{\sqrt{E_{cx} E_{cy}}}{\sqrt{E_{Fx} E_{Fy}}} \left(\frac{E_{oF}}{E_{oc}}\right)^{1/2} \left(\frac{E_{cx}}{E_{Fx}}\right)^{1/2} \frac{\Omega_o}{12\sin^4\theta} \tag{1.90}$$

$$V = \frac{(1-\nu_{xyc}\nu_{yxc})\sigma_F M_o}{\sin^4\theta\cos^2\theta (E_{cx}E_{cy})^{1/2}\Omega_o K} \left(\frac{E_{oc}}{E_{oF}}\right)^{1/2} \left(\frac{E_{Fx}}{E_{cx}}\right)^{1/2}. \tag{1.91}$$

A very rapid iteration produces the value of K.

The actual design procedures are given in summary in Section I of this chapter.

H. Structural Optimization of Panels with Faces and Core
    of the Same Orthotropic Material

The governing equtions with which to optimize for
this construction can be obtained easily by contracting the
set (1.73) through (1.77) by letting $E_{fx} = E_{cx} = E_x$,
$E_{fy} = E_{cy} = E_y$, $\rho_f = \rho_c = \rho$, $\nu_{xyf} = \nu_{xyc} = \nu_{xy}$,
$\nu_{yxf} = \nu_{yxc} = \nu_{yx}$, and $\sigma_f = \sigma_c = \sigma$. The equations
become:

$$N_x = \frac{\pi^2 \sqrt{E_x E_y}\ t_f\, h_c^2\, K}{2b^2} \tag{1.92}$$

$$\sigma = \frac{\pi^2\, E_0\, t_f^2}{12\, h_c^2\, \tan^2\theta\, (1-\nu_{xy}\nu_{yx})} \tag{1.93}$$

$$\sigma = \frac{\pi^2 E_0}{3}\ \frac{t_c^2}{h_c^2}\ \frac{\cos^2\theta}{(1-\nu_{xy}\nu_{yx})} \tag{1.94}$$

$$W - W_{ad} = \rho\left[2t_f + \frac{t_c}{\sin\theta}\right] = N_x\ \frac{\rho}{\sigma} \tag{1.95}$$

$$N_x = \sigma\left[2t_f + \frac{t_c}{\sin\theta}\right] = (W-W_{ad})\frac{\sigma}{\rho} \tag{1.96}$$

Proceeding exactly as before, the weight equation can be expressed in terms of θ only, and becomes,

$$\frac{W-W_{ad}}{b} = \frac{3^{4/4}\rho\left(N_x/b\right)^{1/2}\left(1-\nu_{xy}\nu_{yx}\right)^{1/4}}{K^{1/4}\pi\left(E_xE_y\right)^{1/8}E_0^{1/4}}\frac{\left[1+4\sin^2\theta\right]^{3/4}}{\sin\theta\left(\cos\theta\right)^{1/2}} \tag{1.97}$$

The derivative of (1.97) with respect to θ provides the unique value of θ which will result in a minimum weight structure. It is found to be

$$\sin^2\theta = \frac{2}{7}, \quad \theta \approx 32.4^\circ \tag{1.98}$$

It should be noted that it is independent of the value of the load index, the buckling coefficient K, and the materials used. Note that for the orthotropic construction where faces and core material are the same, the optimum angle θ is identical to that of isotropic construction with the face and core material the same (see Equation (1.27)).

The "universal relationship" relating the "load index" to the unique value of face stress which will result in minimum weight structure is found to be:

$$\frac{N_x}{b} = \frac{45}{2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/2}\sigma^2}{\pi^2K^{1/2}E_0^{1/2}\left(E_xE_y\right)^{1/4}} \tag{1.99}$$

The geometric variables and the weight equation in terms of the stress are determined as:

$$\frac{t_F}{b} = \frac{6\left(1-\nu_{xy}\nu_{yx}\right)^{1/2}\sigma}{\pi^2 K^{1/2} E_0^{1/2}\left(E_x E_y\right)^{1/4}} \tag{1.100}$$

$$\frac{t_G}{b} = 3\left(\frac{1}{2}\right)^{1/2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/2}\sigma}{E_0^{1/2}\left(E_x E_y\right)^{1/4} K^{1/2}} \tag{1.101}$$

$$\frac{h_c}{b} = \frac{1}{\left(E_x E_y\right)^{1/4}}\left(\frac{15}{2}\frac{\sigma}{\pi^2 K}\right)^{1/2} \tag{1.102}$$

$$\frac{W-W_{ad}}{b} = \frac{45}{2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/2}\rho\sigma}{\pi^2 K^{1/2} E_0^{1/2}\left(E_x E_y\right)^{1/4}} \tag{1.103}$$

The same variables in terms of the load index are found to be:

$$\frac{t_F}{b} = 2\left(\frac{2}{5}\right)^{1/2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/4}\left(N_x/b\right)^{1/2}}{K^{1/4}\pi E_0^{1/4}\left(E_x E_y\right)^{1/8}} \tag{1.104}$$

$$\frac{t_G}{b} = \left(\frac{1}{3}\right)^{1/2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/4}\left(N_x/b\right)^{1/2}}{\pi K^{1/4} E_0^{1/4}\left(E_x E_y\right)^{1/8}} \tag{1.105}$$

$$\frac{h_c}{b} = \left(\frac{5}{2}\right)^{1/4}\frac{E_0^{1/4}\left(N_x/b\right)^{1/4}}{K^{1/4}\pi^{1/2}\left(E_x E_y\right)^{3/16}\left(1-\nu_{xy}\nu_{yx}\right)^{1/8}} \tag{1.106}$$

$$\frac{W-W_{ad}}{b} = 3\left(\frac{5}{2}\right)^{1/2}\frac{\left(1-\nu_{xy}\nu_{yx}\right)^{1/4}\rho\left(N_x/b\right)^{1/2}}{K^{1/4}\pi E_0^{1/4}\left(E_x E_y\right)^{1/8}} \tag{1.107}$$

Note also that, as in the isotropic construction in which the face and core material are identical,

$$
\left.
\begin{array}{l}
\dfrac{t_c}{t_f} = \sqrt{\dfrac{7}{8}} \\[2em]
\text{and} \\[0.5em]
\dfrac{W_c}{W_f} = \dfrac{7}{8}
\end{array}
\right\} \qquad (1.108)
$$

Again to find the buckling coefficient K, Figures 3 and 5 of Reference 3 are used, making use of the following parameters,

$$
\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{7}{24} \qquad (1.109)
$$

$$
V = \frac{21}{2} \frac{(1 - \nu_{xy} \nu_{yx}) \sigma}{(E_x E_y)^{1/2} K} \qquad (1.110)
$$

Although one might obtain approximate results for stresses above the proportional limit, due to the nature of the quantities E_0 and $E_x E_y$ involving Poisson's ratio effects, these methods, strictly speaking, should be limited to stresses at or below the proportional limit.

The actual design procedures are given in summary in Section I of this chapter.

I. Design Procedures for Panels of Triangulated, Corrugated
Core (Single Truss Core) Construction Under Uniaxial
Compression

This section presents in an abbreviated form the
steps to follow in determining the minimum weight design
curves for panels of this type of construction.

1. Panels in Which Faces and Cores are of the Same
Isotropic Material

a. Known quantities:
$$\theta = 32.4°; \quad \frac{E_c \overline{I}_c}{E_f \overline{I}_f} = \frac{7}{24}$$

b. Select material, and stress values of interest.
Determine values of reduced modulus $\overline{E}$, where
$\overline{E} = \sqrt{EE_t}$ *, for stress values above the
proportional limit.

c. Using Figure 3 or 5 of NACA TN 2679 determine
value of buckling coefficient K from ordinate,
for $\dfrac{E_c I_c}{E_f \overline{I}_f} = \dfrac{7}{24}$, and an abscissa of V where,
for each σ

$$V = \frac{21(1-v^2)\sigma}{2 \overline{E} K}$$

It is necessary to iterate to find the value of

K, however for this construction V is very small

_____
*
There are several other widely used equations which could be
used to determine the reduced modulus. Since there is no
general agreement as to which is the most accurate, the
simplest form has been suggested here.

over the entire range of stresses and hence X
is nearly the ordinate intercept value.

d. For each stress value selected determine $(N_{x/b})$

$$\frac{N_x}{b} = \frac{45}{2} \frac{(1-\nu^2)^{1/2} \sigma^2}{\pi^2 \bar{E} K^{1/2}}$$

e. Determine $(h_{c/b})$ for each σ

$$\frac{h_c}{b} = \left[\frac{15}{2} \frac{\sigma}{\pi^2 \bar{E} K}\right]^{1/2}$$

f. Determine $t_{f/b}$ for each σ

$$\frac{t_f}{b} = \frac{6(1-\nu^2)^{1/2} \sigma}{\pi^2 \bar{E} K^{1/2}}$$

g. Determine $t_{c/b}$ for each σ

$$\frac{t_c}{b} = \sqrt{\frac{7}{8}}\left(\frac{t_f}{b}\right)$$

h. Determine $(W - W_{ad})/b$ for each σ

$$\frac{W - W_{ad}}{b} = \left(\frac{N_x}{b}\right)\frac{\rho}{\sigma}$$

2. **Panels in Which Faces and Core are of Different**
    Isotropic Materials

The following procedures are applicable for stresses
below the proportional limit of both face and core materials.

a. Select materials and face stress values of interest

b. Determine following relationships

$$\Omega = \left[\frac{1-\nu_c^2}{1-\nu_f^2}\right]^{1/2} , \quad E^* = \frac{E_c}{E_f} \cdot \Omega , \quad \rho^* = \frac{\rho_c}{\rho_f} \cdot \Omega$$

c. Obtain the value of θ from

$$\left[8 + 8\rho^* - 2E^*\right] \sin^4\theta - \left[6\rho^* - 4E^* - \tfrac{1}{2}\rho^* E^*\right] \sin^2\theta - \rho^* E^* = 0$$

d. Determine $E_c \bar{I}_c / E_f \bar{I}_f$

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{E_c}{E_f} \frac{\Omega}{12 \sin^2\theta}$$

e. Determine value of K from Figure 3 or 5 of NACA
TN 2679 for each σ where abscissa V is

$$V = \frac{(1 - \nu_c^2) \sigma \left[E^* + 4 \sin^2\theta\right]}{E_c \Omega K \sin^2\theta \cos^2\theta}$$

Again an iteration is necessary

f. Determine N_x/b for each stress value of interest

$$\frac{N_x}{b} = \frac{\sqrt{3}(1 - \nu_f^2)^{1/2} \sigma_f^2 \left[E^* + 4 \sin^2\theta\right]^{3/2}}{\pi^2 K^{1/2} E_f \sin^2\theta \cos\theta}$$

g. Determine h_c/b for each σ_f

$$\frac{h_c}{b} = \left\{\frac{\left[E^* + 4 \sin^2\theta\right] \sigma_f}{\pi^2 E_f K \sin^2\theta}\right\}^{1/2}$$

h. Determine t_f/b for each σ_f

$$\frac{t_f}{b} = \frac{2\sqrt{3}(1 - \nu_f^2)^{1/2} \sigma_f \left[E^* + 4 \sin^2\theta\right]^{1/2}}{\pi^2 E_f K^{1/2} \cos\theta}$$

i. Determine t_c/b for each σ_f

$$\frac{t_c}{b} = \frac{\Omega}{2 \sin\theta}\left(\frac{t_f}{b}\right)$$

j. Determine $(W - W_{ad})/b$ for each σ_f

$$\frac{W - W_{ad}}{b} = \frac{\sqrt{3}(1 - \nu_f^2)^{1/2} \rho_f \sigma_f \left[\rho^* + 4 \sin^2\theta\right]\left[E^* + 4 \sin^2\theta\right]^{1/2}}{\pi^2 E_f K^{1/2} \sin^2\theta \cos\theta}$$

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k. Determine σ_c for each σ_f to insure that both stresses are below the proportional limit.

$$\sigma_c = \sigma_f \frac{E_c}{E_f}$$

3. **Panels in Which Faces and Core are of the Same Orthotropic Material**

The following procedures apply to stresses below the proportional limit of the material.

a. Known quantities $\theta = 32.4^\circ$ ($\sin^2\theta = \frac{2}{7}$, $\cos^2\theta = \frac{5}{7}$)

$$E_c\overline{I}_c/E_f\overline{I}_f = 7/24$$

b. Select material and stress values of interest

c. Calculate E_0

$$E_0 = \frac{1}{2}\left[\sqrt{E_x E_y} + \nu_{yx} E_x + 2 G_{xy}\left(1 - \nu_{xy}\nu_{yx}\right)\right]$$

d. Using Figure 3 or 5 of NACA TN 2679 determine value of K for $E_c\overline{I}_c = 7/24$, and an abscissa value of a/b for each σ

$$\nabla = \frac{21\left(1 - \nu_{xy}\nu_{yx}\right)\sigma}{\left(E_x E_y\right)^{1/2} K}$$

An iteration is necessary.

e. For each σ determine $N_{x/b}$

$$\frac{N_x}{b} = \frac{45}{2}\frac{\left(1 - \nu_{xy}\nu_{yx}\right)^{1/2}\sigma^2}{\pi^2 K^{1/2} E_0^{1/2}\left(E_x E_y\right)^{1/2}}$$

f. Determine h_c/b for each σ

$$\frac{h_c}{b} = \frac{1}{(E_x E_y)^{1/4}} \left[\frac{15}{2} \frac{\sigma}{\pi^2 K} \right]^{1/2}$$

g. Determine t_f/b for each σ

$$\frac{t_f}{b} = \frac{6(1 - \nu_{xy} \nu_{yx})^{1/2} \sigma}{\pi^2 K^{1/2} E_o^{1/2} (E_x E_y)^{1/4}}$$

h. Determine t_c/b for each σ

$$\frac{t_c}{b} = \sqrt{\frac{7}{8}} \left(\frac{t_f}{b} \right)$$

i. Determine $\dfrac{W - W_{ad}}{b}$ for each σ

$$\frac{W - W_{ad}}{b} = \left(\frac{N_x}{b} \right) \frac{\rho}{\sigma}$$

4. **Panels in Which Faces and Core are of Different Orthotropic Materials**

The following procedures are applicable to stresses below the proportional limit of both face and core materials.

   a. Select materials and face stress values of interest

   b. Determine the following relationships

$$E_{oi} = \frac{1}{2} \left[\sqrt{E_{ix} E_{iy}} + \nu_{yxi} E_{ix} + 2G_{xyi} (1 - \nu_{xyi} \nu_{yxi}) \right]$$

$$i = (c, f)$$

$$\Omega_o = \left[\frac{1 - \nu_{xyc} \nu_{yxc}}{1 - \nu_{xyf} \nu_{yxf}} \right]^{1/2}$$

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c.   Obtain θ from the following

$$\left\{ 8 + 8\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{cx}}{E_{fx}}\right)^{\!1/2}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\Omega_o - 2\left(\frac{E_{cx}}{E_{fx}}\right)^{\!1/2}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\Omega_o \right\} \, Sin^4\theta$$

$$-\left\{ 6\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{cx}}{E_{fx}}\right)^{\!1/2}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\Omega_o - 4\left(\frac{E_{cx}}{E_{fx}}\right)^{\!1/2}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\Omega_o \right.$$

$$\left. -\frac{3}{2}\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{cx}}{E_{fx}}\right)^{2}\left(\frac{E_{of}}{E_{oc}}\right)\Omega_o^{\,2} \right\} \, Sin^2\theta$$

$$-\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{cx}}{E_{fx}}\right)^{2}\left(\frac{E_{of}}{E_{oc}}\right)\Omega_o^{\,2} = 0$$

d.   Determine $E_c \bar{I}_c / E_f \bar{I}_f$

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{\sqrt{E_{cx}E_{cy}}}{\sqrt{E_{fx}E_{fy}}}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\left(\frac{E_{cx}}{E_{fx}}\right)^{\!1/2}\frac{\Omega_o}{12\,Sin^4\theta}$$

e.   Solve for M_o

$$M_o = 4\,Sin^2\theta + \left(\frac{E_{cx}}{E_{fx}}\right)^{\!3/2}\left(\frac{E_{of}}{E_{oc}}\right)^{\!1/2}\Omega_o$$

f.   Determine K from Figure 3 or 5 of NACA TN 2679 for $E_c\bar{I}_c / E_f\bar{I}_f$ calculated above and the abscissa V given by

$$V = \frac{\left(1 - \nu_{xyc}\,\nu_{yxc}\right)\sigma_F\,M_o}{Sin^2\theta\,Cos^2\theta\left(E_{cx}E_{cy}\right)^{1/2}\Omega_o K}\left(\frac{E_{oc}}{E_{of}}\right)^{\!1/2}\left(\frac{E_{fx}}{E_{cx}}\right)^{\!1/2}$$

g. Determine N_x/b for each σ_f

$$\frac{N_x}{b} = \frac{\sqrt{3}\left(1-\nu_{xyf}\,\nu_{yxf}\right)^{1/2} M_o^{3/2} \sigma_F^2}{\pi^2 K^{1/2} E_{of}^{1/2} \left(E_{fx} E_{fy}\right)^{1/4} \sin^2\theta \cos\theta}$$

h. Determine h_c/b for each σ_f

$$\frac{h_c}{b} = \frac{M_o^{1/2}\,\sigma_f^{1/2}}{K^{1/2}\,\pi\,\left(E_{fx}E_{fy}\right)^{1/4}\sin\theta}$$

i. Determine t_f/b for each σ_f

$$\frac{t_f}{b} = \frac{2\sqrt{3}\left(1-\nu_{xyf}\,\nu_{yxf}\right)^{1/2} M_o^{1/2}\sigma_f}{\pi^2 E_{of}^{1/2}\left(E_{fx}E_{fy}\right)^{1/4} K^{1/2}\cos\theta}$$

j. Determine t_c/b for each σ_f

$$\frac{t_c}{b} = \left(\frac{E_{of}}{E_{oc}}\right)^{1/2}\left(\frac{E_{cx}}{E_{fx}}\right)^{1/2}\frac{\Omega_o}{2\sin\theta}\left(\frac{t_f}{b}\right)$$

k. Determine $(W - W_{ad})/b$ for each σ_f

$$\frac{W - W_{ad}}{b} = \left(\frac{N_x}{b}\right)\frac{\left\{4\sin^2\theta + \left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{cx}}{E_{fx}}\right)^{1/2}\left(\frac{E_{of}}{E_{oc}}\right)^{1/2}\Omega_o\right\}}{M_o}\left(\frac{\rho_f}{\sigma_f}\right)$$

l. Determine σ_c for each σ_f to insure it remains below proportional limit

$$\sigma_c = \sigma_f\,\frac{E_{cx}}{E_{fx}}$$

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CHAPTER 2.

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT WEB-CORE SANDWICH PANELS SUBJECTED TO UNIAXIAL COMPRESSION

A. Introduction

Consider a flat web-core sandwich panel, generalized to include some arbitrary angle Θ, as shown in Figure 4 below.
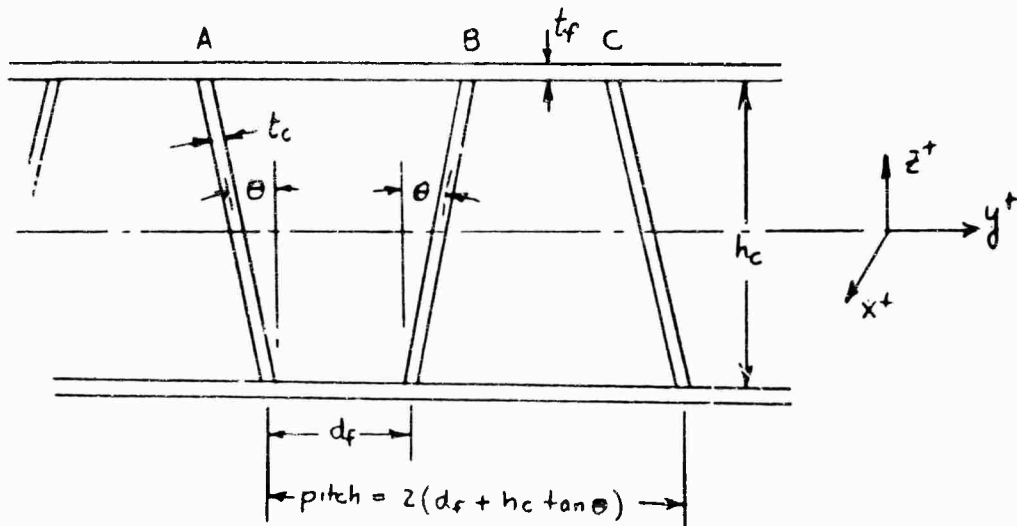

Figure 4

Generalized Web-Core Sandwich Panel

The overall panel geometry is given in Figure 2.

There are five geometric variables with which to optimize; namely, the core depth (h_c), the web thickness (t_c), the face thickness (t_f), the angle the web makes with a line normal to the faces (Θ), and the distance between web elements (d_f).

The panel is considered to fail if any of the following instabilities occur: overall instability, local face buckling in the region from A to B; local face buckling in the region B to C; and web element buckling. Hence there are five geometric variables and four modes of instability. To describe the instability mathematically, the analytical expression used in each case is the best variable from the literature.

B. Elastic and Geometric Constants Associated with Web-Core Construction

The elastic and geometric constants for the web-core construction can be determined from those given in more general form by Libove and Hubka in Reference 1.

The area of the core per unit width of corrugative crossection parallel to the yz plane, $\overline{A}_c$, is given by

$$\overline{A}_c = \frac{t_c h_c}{(d_f + h_c \tan\theta) \cos\theta} \qquad \text{(in.)} \qquad (2.1)$$

The moment of inertia of the core per unit width of corrugation crossection parallel to the y-z plane taken about the centroidal axis of the corrugation crossection, $\overline{I}_c$ is seen to be

$$\overline{I}_c = \frac{t_c h_c^3}{12 \cos\theta (d_f + h_c \tan\theta)} - \frac{\overline{A}_c h_c^2}{12} \qquad \text{(in.}^3) \qquad (2.2)$$

The extensional stiffness of the panel in the x-direction EA_x is given by

$$EA_x = E_c \overline{A}_c + 2 E_f t_f \qquad \text{(lbs./in.)} \qquad (2.3)$$

where E_c and E_f are the compressive modulus of elasticity of the core and face material respectively.

Neglecting the shear stiffness of the facings, the transverse shear stiffness, per unit width in the x direction, of an element of the sandwich cut by two y-z planes is seen to be

$$D_{q_y} = 0 \qquad\qquad (2.4)$$

since there is no shear continuity in the y direction for the
web-core construction. Hence the transverse shear flexibility
parameter in the y direction, V, is given by

$$V = \infty . \qquad\qquad (2.5)$$

Lastly, the moment of inertia per unit width, $\overline{I}_f$, of
the faces, considered as membranes, with respect to the sandwich
plate middle surface, is seen to be

$$\overline{I}_f = \frac{t_f h_c^2}{2} \qquad (in.^3) \qquad\qquad (2.6)$$

Since $t_f \ll h_c$, the core depth, h_c, can be taken as the
distance between the centerlines of the faces.

C. Governing Equations for Panels Composed of Orthotropic Materials

Because in Chapter 1, the governing equations were formulated rigorously for panels with isotropic materials, and there the philosophy and assumptions were discussed in extending those expressions to panels composed of orthotropic materials, the details will r t be repeated. The most general case is the panel whose face and core are composed of different orthotropic materials, as given below. For isotropic materials, or the same material used in face and core, the expression can be easily simplified.

1. Overall Instability

The expression is identical to the one used for triangulated core construction in Chapter 1, Equation 1.63, and is repeated below:

$$N_x = \frac{\pi^2 (E_{fx} E_{fy})^{1/2} t_f h_c^2}{2b^2} K \tag{2.7}$$

In this case K, the buckling coefficient, is given in Figure 2 of Reference 3 for panels with edges simply supported, and by Figure 4 of Reference 3 for panels with edges clamped. Unlike the case of triangulated core construction where V was finite, in this construction V is infinite. The result is that where for triangulated core construction K became constant for a $>$ b, for web core construction K is always a function of a/b.

To utilize Figures 2 and 4, or for more accuracy to utilize the general expression for K given as Equation 6 of Reference 3, it is necessary to determine the ratio of bending stiffness of the core in the x direction to the bending stiffness of the faces. Utilizing Equations (2.2) and (2.6) the expression can be written as:

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{(E_{cx}E_{cy})^{1/2} t_c h_c}{6(E_{fx}E_{fy})^{1/2} t_f (d_f + h_s \tan\theta) \cos\theta} \qquad (2.8)$$

2. Face Plate Instability

Using Equation (1.67), the expression for face buckling in the region of A to B of Figure 4 is found by setting $b = d_f + 2h_s \tan\theta$, $h = t_f$, $E_o = E_{of}$, $\nu = \nu_f$, and $\sigma = \sigma_f$. The resulting expression is:

$$\sigma_f = \frac{\pi^2 E_{of} t_f^2}{3(1-\nu_{xyf} \nu_{yxf})(d_f + 2h_c \tan\theta)^2} \qquad (2.9)$$

For the face panel buckling in the region B to C of Figure 4, substituting into Equation (1.67) the following: $b = d_f$, $h = t_f$, $E_o = E_{of}$, $\nu = \nu_f$ and $\sigma = \sigma_f$, the following expression is obtained:

$$\sigma_f = \frac{\pi^2 E_{of} t_f^2}{3(1-\nu_{xyf} \nu_{yxf}) d_f^2} \qquad (2.10)$$

3. Web Plate Instability

Again using Equation (1.67), if $b = h_c/\cos\theta$, $h = t_c$,

$R_o = R_{o_c}$, $\nu = \nu_c$, and $\sigma = \sigma_c$, the expression is seen to be:

$$\sigma_c = \frac{\pi^2 E_{oc} t_c^2 \cos^2\theta}{3(1-\nu_{xyc}\nu_{yxc}) h_c^2} \tag{2.11}$$

4. Load-Stress Relationship

Analogous to the development in Chapter 1, the face stress can be related to the stress in the web elements by

$$\sigma_c = \frac{E_{cx}}{E_{fx}} \sigma_f, \tag{2.12}$$

and the load per inch, N_x, is related to the face stress by

$$N_x = \sigma_f \left[\frac{E_{cx}}{E_{fx}} \frac{t_c h_c}{(d_f + h_c \tan\theta)\cos\theta} + 2t_f \right]. \tag{2.13}$$

Again, it should be noted that where the face material differs from the core material, the expressions (2.12) and (2.13) pertain only to the range of loads and stresses less than the proportional limit of both the face and core material. When either face or core material are stressed beyond the proportional limit an iterative procedure must be employed utilizing the properties described in Figure 3.

5. Weight Equation

The weight equation, analogous to the procedures of Chapter 1, is found to be

$$W - W_{ad} = 2\rho_f t_f + \frac{\rho_c t_c h_c}{(d_f + h_c \tan\theta)\cos\theta}. \tag{2.14}$$

D. <u>Structural Optimization of Panels with Faces and Core of</u>
   <u>Different Orthotropic Materials</u>

The important relationships to utilize in the optimiza-
tion are the stability expressions given in Equations (2.7),
(2.9), (2.10), (2.11); and the load-stress relationships given
by Equations (2.12) and (2.13). Thus there are six equations,
and the seven unknowns; t_f, h_c, t_c, θ, d_f, σ_f, and σ_c. The
seventh equation is obtained by expressing the weight equation
(2.14) in suitable form, and setting the derivative of the
equation with respect to a convenient variable equal to zero,
to obtain the value of that variable which will insure minimum
weight.

The philosophy of optimization is identical to that
expressed in Sections D and E of Chapter 1; namely, that all
independent modes of failure that cause the panel to be con-
sidered structurally unsound must occur simultaneously to produce
a panel of minimum weight.

Before proceeding it is interesting to note that setting
$d_f = 0$ in the equations of Section C above reduce them to the
same set given in Section G, Chapter 1 for the triangulated
construction. Obviously, with $d_f = 0$, Equation (2.10) does
not exist

At the outset, equating Equations (2.9) to (2.10) results
in the fact that for optimum construction of web-core panels.

$$\theta = 0. \tag{2.15}$$

This is intuitively obvious. The result is that the expressions with which to optimize are simplified, and the construction given in Figure 4 can now be replaced by the familiar web-core construction.

Proceeding as before, the final expressions for the optimized construction are written as follows.

Universal Relationship

$$\frac{N_x}{b} = \frac{\sqrt{6}\,(1-\nu_{xyf}\nu_{yxf})^{1/4}(1-\nu_{xyc}\nu_{yxc})^{1/4}}{(E_{of}E_{oc})^{1/4}(E_{fx}E_{fy})^{1/4}\,\pi^2 K^{1/2}}\left(\frac{E_{cx}}{E_{fx}}\right)^{7/4}\left(\frac{\rho_f}{\rho_c}\right)R^{3/2}\sigma_f^2 \tag{2.16}$$

where K is determined from Figures 2 or 4 of Reference 3, or Equation 6 of Reference 3, in which,

$$V = \frac{\pi^2 E_f \bar{I}_f}{b^2 D_{yy}} = \infty \tag{2.17}$$

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{(E_{cx}E_{cy})^{1/2}}{6(E_{fx}E_{fy})^{1/2}}\frac{t_c h_c}{t_f d_f} = \frac{1}{6}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}E_{cy}}{E_{fx}E_{fy}}\right)^{1/2} \tag{2.18}$$

It is also convenient to define a parameter R as:

$$R = 1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_{fx}}{E_{cx}}\right) \tag{2.19}$$

The optimized geometric variables and the optimized weight in terms of the optimum face stress σ_f, and the load index (N_x/b), are given below.

$$\frac{h_c}{b} = \left\{2\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}}{E_{fx}}\right)\frac{R\,\sigma_f}{(E_{fx}E_{fy})^{1/2}\pi^2 K}\right\}^{1/2}$$

$$= \left\{\frac{2^{3/4}}{3^{1/4}\pi K^{3/4}}\left(\frac{\rho_f}{\rho_c}\right)^{1/2}\left[\left(\frac{E_{cx}}{E_{fx}}\right)\frac{E_{of}E_{oc}}{(1-\nu_{xyf}\nu_{yxf})(1-\nu_{xyc}\nu_{yxc})}\right]^{1/8}\frac{R^{1/4}(N_x/b)^{1/2}}{(E_{fx}E_{fy})^{3/8}}\right\}^{1/2} \tag{2.20}$$

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$$\frac{d_f}{b} = \frac{2^{1/2}}{\pi K^{1/2}} \left[\frac{1-\nu_{xyc}\nu_{yxc}}{1-\nu_{xyf}\nu_{yxf}}\right]^{1/4} \left(\frac{E_{yf}}{E_{oc}}\right)^{1/4} \left(\frac{E_{cx}}{E_{fx}}\right)^{3/4} \frac{R^{1/2}}{(E_{ox}E_{cy})^{1/4}} \sigma_f^{1/2}$$

$$= \frac{2^{7/8}}{3^{3/8}\pi^{1/4}K^{3/8}} \frac{(1-\nu_{xyc}\nu_{yxc})^{3/16}}{(1-\nu_{xyf}\nu_{yxf})^{5/16}} \left(\frac{E_{of}}{E_{oc}}\right)^{1/4} \left(\frac{E_{cx}}{E_{fx}}\right)^{5/8} \left(\frac{\rho_c}{\rho_f}\right)^{1/4} \frac{(E_{of}E_{oc})^{1/4}}{(E_{fx}E_{cy})^{3/8}} R^{1/4}(N_x/b)^{1/4} \quad (2.21)$$

$$\frac{t_f}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}} \left[\frac{(1-\nu_{xyf}\nu_{yxf})(1-\nu_{xyc}\nu_{yxc})}{(E_{fx}E_{fy})(E_{of}E_{oc})}\right]^{1/4} \left(\frac{E_{cy}}{E_{fx}}\right)^{3/4} R^{1/2} \sigma_f$$

$$= \frac{6^{3/4}}{\pi K^{3/4}} \left[\frac{(1-\nu_{xyf}\nu_{yxf})(1-\nu_{xyc}\nu_{yxc})}{(E_{fx}E_{fy})(E_{of}E_{oc})} \left(\frac{E_{fx}}{E_{cx}}\right)\right]^{1/8} \left(\frac{\rho_c}{\rho_f}\right)^{1/2} \frac{(N_x/b)^{1/2}}{R^{1/4}} \quad (2.22)$$

$$\frac{t_c}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}} \frac{(1-\nu_{xyc}\nu_{yxc})^{1/2}}{E_{oc}^{1/2}[E_{fx}E_{fy}]^{1/4}} \left(\frac{\rho_f}{\rho_c}\right)^{1/2} \left(\frac{E_{cx}}{E_{fx}}\right) R^{1/2} \sigma_f$$

$$= \frac{6^{3/4}}{\pi K^{3/4}} \frac{(1-\nu_{xyc}\nu_{yxc})^{3/4}}{(1-\nu_{xyf}\nu_{yxf})^{1/8}} \left[\frac{E_{of}E_{oc}}{E_{fx}E_{fy}}\right]^{1/8} \frac{1}{E_{oc}^{1/2}} \left(\frac{E_{cx}}{E_{fx}}\right)^{1/4} \frac{(N_x/b)^{1/2}}{R^{1/4}} \quad (2.23)$$

$$\frac{W-W_{ad}}{b} = \frac{3\sqrt{6}\,\rho_f}{\pi^2 K^{1/2}} \left[\frac{(1-\nu_{xyf}\nu_{yxf})(1-\nu_{xyc}\nu_{yxc})}{(E_{fx}E_{fy})(E_{of}E_{oc})}\right]^{1/4} \left(\frac{E_{cy}}{E_{fx}}\right)^{3/4} R^{1/2} \sigma_f$$

$$= \frac{3(6)^{1/4}(\rho_c\rho_f)^{1/2}}{\pi K^{3/4}} \left[\frac{(1-\nu_{xyf}\nu_{yxf})(1-\nu_{xyc}\nu_{yxc})}{(E_{fx}E_{fy})(E_{of}E_{oc})}\right]^{1/8} \frac{(N_x/b)^{1/2}}{R^{1/4}} \quad (2.24)$$

From these expressions, some interesting relationships are derived, namely,

$$\frac{h_c}{d_f} = \left[\left(\frac{\rho_f}{\rho_c}\right)^2 \left(\frac{1-\nu_{xyf}\nu_{yxf}}{1-\nu_{xyc}\nu_{yxc}}\right) \left(\frac{E_{oc}}{E_{of}}\right) \left(\frac{E_{fx}}{E_{cx}}\right)\right]^{1/4} \quad (2.25)$$

$$\frac{t_c}{t_f} = \left[\left(\frac{\rho_f}{\rho_c}\right)^2 \left(\frac{1-\nu_{xyc}\nu_{yxc}}{1-\nu_{xyf}\nu_{yxf}}\right) \left(\frac{E_{of}}{E_{oc}}\right) \left(\frac{E_{cx}}{E_{fx}}\right)\right]^{1/4} \quad (2.26)$$

$$\frac{h_c}{d_f} = \left(\frac{\rho_f}{\rho_c}\right) \left(\frac{t_f}{t_c}\right) \quad (2.27)$$

Thus in the general case of orthotropic materials, and different face and core materials, the ratios of (h_c/d_f) and (t_c/t_f) as well as the relationship between these two ratios are independent of load, and independent of boundary conditions for _optimum_ construction.

It can also be shown that even in this general material system, the ratio of face weight to core weight for _optimum_ construction is:

$$\frac{W_f}{W_c} = 2 .$$

(2.28)

This ratio too is independent of load index or boundary conditions, and also results in the fact that

$$\frac{W - W_{ad}}{b} = 3\rho_f\left(\frac{t_f}{b}\right) .$$

(2.28a)

E. **Structural Optimization of Panels with Faces and Core of Different Isotropic Materials**

The resulting expressions for the optimized structure can be obtained from Equations (2.16) through (2.28) by allowing $E_{fx} = E_{fy} = E_{of} = E_f$, $E_{cx} = E_{oc} = E_c$, and $\nu_{xy} = \nu_{yx} = \nu$ for both face and core material. The results are:

Universal Relationship

$$\frac{N_x}{b} = \frac{\sqrt{6}\,(1-\nu_f^2)^{1/4}(1-\nu_c^2)^{1/4}}{\pi^2 K^{1/2}} \frac{E_c^{3/2}}{E_f^{5/2}} \left(\frac{\rho_f}{\rho_c}\right) R^{3/2} \sigma_f^2, \qquad (2.29)$$

where K is determined from Figure 2 or 4 of Reference 3, or from Equation 6 of Reference 3 in which

$$\nabla = \infty \qquad (2.30)$$

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{1}{6}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_c}{E_f}\right) \qquad (2.31)$$

For this case R becomes

$$R = 1 + 2\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_f}{E_c}\right) \qquad (2.32)$$

The optimized geometric variables as well as the minimum weight expression are given in terms of the optimum face stress σ_f and the load index (N_x/b).

$$\frac{h_s}{b} = \left\{\frac{2}{\pi^2 K}\left(\frac{\rho_f}{\rho_c}\right)\frac{E_c}{E_f^2}\, R\,\sigma_f\right\}^{1/2}$$

$$= \left\{\frac{2^{3/4}}{3^{1/4}\pi K^{3/4}}\left(\frac{\rho_f}{\rho_c}\right)^{1/2}\frac{E_c^{1/4}}{E_f^{3/4}}\frac{R^{1/4}(N_x/b)^{1/2}}{(1-\nu_f^2)^{1/8}(1-\nu_c^2)^{1/8}}\right\}^{1/2} \qquad (2.33)$$

$$\frac{d_F}{b} = \frac{2^{1/2}}{\pi K^{1/2}} \left(\frac{1-\nu_c^2}{1-\nu_f^2}\right)^{1/4} \frac{E_c^{1/2}}{E_F} R^{1/2} \sigma_f^{1/2}$$

$$= \frac{2^{3/8}}{3^{1/8} \pi^{1/2} K^{3/8}} \frac{(1-\nu_c^2)^{3/16}}{(1-\nu_f^2)^{5/16}} \left(\frac{\rho_c}{\rho_f}\right)^{1/4} \frac{E_c^{1/8}}{E_f^{3/8}} R^{1/8} (N_x/b)^{1/4} \tag{2.34}$$

$$\frac{t_F}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}} (1-\nu_f^2)^{1/4} (1-\nu_c^2)^{1/4} \frac{E_c^{1/2}}{E_f^{1/2}} R^{1/2} \sigma$$

$$= \frac{6^{1/4}}{\pi K^{1/4}} \frac{(1-\nu_c^2)^{1/8} (1-\nu_f^2)^{1/8}}{E_f^{1/4} E_c^{1/8}} \left(\frac{\rho_c}{\rho_f}\right)^{1/2} \frac{(N_x/b)^{1/2}}{R^{1/4}} \tag{2.35}$$

$$\frac{t_c}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}} (1-\nu_c^2)^{1/2} \frac{E_c^{1/2}}{E_f^{1/2}} \left(\frac{\rho_f}{\rho_c}\right)^{1/2} R^{1/2} \sigma$$

$$= \frac{6^{1/4}}{\pi K^{1/4}} \frac{(1-\nu_c^2)^{3/8}}{(1-\nu_f^2)^{1/8}} \frac{(N_x/b)^{1/2}}{E_f^{1/8} E_c^{3/8} R^{1/4}} \tag{2.36}$$

$$\frac{W-W_{ad}}{b} = \frac{3\sqrt{6} \, \rho_F}{\pi^2 K^{1/2}} (1-\nu_f^2)^{1/4} (1-\nu_c^2)^{1/4} \frac{E_c^{1/2}}{E_f^{1/2}} R^{1/2} \sigma_F$$

$$= \frac{3 (6)^{1/4} (\rho_c \rho_F)^{1/2}}{\pi K^{1/4}} \frac{(1-\nu_c^2)^{1/8} (1-\nu_f^2)^{1/8}}{E_f^{3/8} E_c^{1/8}} \frac{(N_x/b)^{1/2}}{R^{1/4}} \tag{2.37}$$

From these expressions several useful relationships can be derived, namely,

$$\frac{h_c}{d_f} = \left(\frac{\rho_f}{\rho_c}\right)^{1/2} \left(\frac{1-\nu_f^2}{1-\nu_c^2}\right)^{1/4} \tag{2.38}$$

$$\frac{t_c}{t_f} = \left(\frac{\rho_f}{\rho_c}\right)^{1/2} \left(\frac{1-\nu_c^2}{1-\nu_f^2}\right)^{1/4} \tag{2.39}$$

$$\frac{h_c}{d_f} = \left(\frac{\rho_F}{\rho_c}\right) \left(\frac{t_f}{t_c}\right) \tag{2.40}$$

$$\tag{2.41}$$

Note all of these expressions for an optimum construction are independent of boundary conditions and load.

F. Structural Optimization of Panels with Faces and Core of the Same Orthotropic Materials

The resulting expressions for this optimized construction can be obtained from Equations (2.16) through (2.28) by simply letting each quantity $(\ )_c = (\ )_f = (\ )$. The results are:

Universal Relationship

$$\frac{N_x}{b} = \frac{9\sqrt{2}\ (1-\nu_{xy}\nu_{yx})^{1/2}\ \sigma_x^2}{\pi^2 K^{1/2} E_0^{1/2} (E_x E_y)^{1/4}} = \left(\frac{W-W_{ad}}{b}\right)\frac{\sigma_x}{\rho} \qquad (2.42)$$

where K is determined from Figure 2 or 4 of Reference 3, or from Equation 6 of Reference 3, in which

$$\nabla = \infty \qquad (2.43)$$

and

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{1}{6} . \qquad (2.44)$$

The optimum geometric variables as well as the expressions for minimum weight, given in terms of the stress, σ (since $\sigma_c = \sigma_f$) or the load index (N_x/b) are given by:

$$\frac{h_f}{b} = \frac{d_f}{b} = \left[\frac{6\sigma_x}{\pi^2 K(E_x E_y)^{1/2}}\right]^{1/2} = \left\{\frac{2^{3/4}}{\pi K^{3/4}}\frac{E_0^{1/4}}{(1-\nu_{xy}\nu_{yx})^{1/4}}\frac{(N_x/b)^{1/2}}{(E_x E_y)^{3/8}}\right\}^{1/2} \qquad (2.45)$$

$$\frac{t_f}{b} = \frac{t_s}{b} = \frac{3\sqrt{2}\ (1-\nu_{xy}\nu_{yx})^{1/2}\sigma_x}{\pi^2 K^{1/2}(E_x E_y)^{1/4} E_0^{1/2}} = \frac{2^{1/4}}{\pi K^{1/4}}\frac{(1-\nu_{xy}\nu_{yx})^{1/4}}{E_0^{1/4}(E_x E_y)^{1/8}}(N_x/b)^{1/2} \qquad (2.46)$$

$$\frac{W - W_{ad}}{b} = \frac{9\sqrt{2}\,\rho}{\pi^2 K^{1/2}} \frac{(1 - \nu_{xy}\nu_{yx})^{1/2}\,\sigma_x}{E_0^{1/2}(E_x E_y)^{1/4}} = \frac{3(2)^{1/4}\,\rho\,(1 - \nu_{xy}\nu_{yx})^{1/4}(N_x/b)^{1/2}}{\pi K^{1/4}\,E_0^{1/4}(E_x E_y)^{1/8}}. \quad (2.47)$$

It is seen that for a panel in which the same orthotropic material is used in face and core (and obviously the stiffer direction placed in the x direction) for minimum weight construction, the face thickness and web thickness are equal ($t_c = t_f$), and each "cell" is square ($h_c = d_f$). Again the facing weight is twice the core weight ($W_f : W_c = 2$). Then since $W_f = 2\rho t_f$, and for this material system $t_c = t_f$, it is seen that

$$\frac{W - W_{ad}}{b} = 3\rho\left(\frac{t_f}{b}\right) = 3\rho\left(\frac{t_y}{b}\right). \qquad (2.48)$$

G. Structural Optimization of Panels with Faces and Core of the Same Isotropic Material

The expressions can be easily deduced from either Equations 2.42 through 2.48 by letting $E_o = E_x = E_y = E$, and $\nu_{xy} = \nu_{yx} = \nu$; or from Equations 2.29 through 2.41 by letting $(\ )_c = (\ )_f = (\ )$.

In this construction, since (1) the material is isotropic and (2) the face stress equals the core stress $(\sigma_c = \sigma_f = \sigma)$, the expressions can be employed for loads resulting in stresses greater than the proportional limit by utilizing a suitably defined reduction factor η , such that $\bar{E} = \eta E$. Thus, in the following $\bar{E}$ is used to denote that the expressions are valid in the range of inelastic deformations.

Universal Relationship

$$\frac{N_x}{b} = \frac{9\sqrt{2}\,(1-\nu^2)^{1/2}\,\sigma^2}{\pi^2\,\bar{E}\,K^{1/2}} = \frac{(W-W_{ad})}{b}\,\frac{\sigma}{\rho} \,. \tag{2.49}$$

The geometric variables and the minimum weight expression can be written as:

$$\frac{h_c}{b} = \frac{d_f}{b} = \left[\frac{6\,\sigma}{\pi^2 K \bar{E}}\right]^{1/2} = \frac{2^{3/8}}{\pi^{1/2}\,k^{3/8}\,(1-\nu_c^2)^{1/8}}\,\frac{(N_x/b)^{1/4}}{\bar{E}^{1/4}} \tag{2.50}$$

$$\frac{t_f}{b} = \frac{t_c}{b} = \frac{3\sqrt{2}\,(1-\nu^2)^{1/2}}{\pi^2 K^{1/2}}\,\frac{\sigma}{\bar{E}} = \frac{2^{1/4}(1-\nu^2)^{1/4}}{\pi K^{1/4}}\,\frac{(N_x/b)^{1/2}}{\bar{E}^{1/2}} \tag{2.51}$$

$$\frac{W-W_{ad}}{b} = \frac{9\sqrt{2}\,\rho(1-\nu^2)^{1/2}}{\pi^2 K^{1/2}}\,\frac{\sigma}{\bar{E}} = \frac{3(2)^{1/4}(1-\nu^2)^{1/4}}{\pi K^{1/4}\,\bar{E}^{1/2}}\left(\frac{N_x}{b}\right)^{1/2}$$
$$= \left(\frac{N_x}{b}\right)\frac{\rho}{\sigma} = 3\rho\left(\frac{t_f}{b}\right) = 3\rho\left(\frac{t_c}{b}\right). \tag{2.52}$$

X. Design Procedures for Minimum Weight Panels of Web-Core Construction Under Uniaxial Compression

This section presents in an abbreviated form the steps to follow in determining the minimum weight design curves for panels with this type of construction.

1. **Panels in Which Faces and Core are of the Same Isotropic Material**

   a. Known quantities:
   $$\theta = 0^\circ, \quad \frac{E_c \overline{I}_c}{E_f \overline{I}_f} = \frac{1}{6}, \quad V = \infty, \quad a/b$$

   b. Select material, and the stress values of interest. Determine values of the reduced modulus $\overline{E}$, where $\overline{E} = \sqrt{EE_t}$ (or equivalent), for stresses above the proportional limit.

   c. Using Figure 2 or 4 of NACA TN 2679 determine the buckling coefficient K from the ordinate for the value of a/b given.

   d. For each value of stress value selected, determine N_x/b by
   $$\frac{N_x}{b} = \frac{9\sqrt{2}\left(1 - \nu^2\right)^{1/2} \sigma^2}{\pi^2 \overline{E} \, K^{1/2}}$$

   e. Determine $\dfrac{h_c}{b}$ and $\dfrac{d_f}{b}$ for each stress value by
   $$\frac{h_c}{b} = \frac{d_f}{b} = \left[\frac{6\sigma}{\pi^2 K \overline{E}}\right]^{1/2}$$

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f. Determine $\dfrac{t_c}{b}$ and $\dfrac{d_f}{b}$ for each stress value by

$$\frac{t_f}{b} = \frac{t_c}{b} = \frac{3\sqrt{2}\,(1-\nu^2)^{1/2}\,\sigma}{\pi^2\,K^{1/2}\,\bar{E}}$$

g. Determine the minimum weight for each σ by

$$\frac{W-W_{ad}}{b} = \left(\frac{N_x}{b}\right)\frac{\rho}{\sigma}$$

2. **Panels in Which Faces and Core are of the Same Orthotropic Material**

The following procedures apply to stresses below the proportional limit of the material.

   a. Known quantities:

$$\phi = 0°, \quad \frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{1}{6}, \quad V = \infty \quad , \quad a/b$$

   b. Select material and stress values of interest.

   c. Using Figures 2 or 4 of NACA TN 2679 determine the buckling coefficient K (the ordinate) for the value of a/b for the panel, or use Equation 6 of NACA TN 2679.

   d. Calculate E_o

$$E_o = \frac{1}{2}\left[\sqrt{E_x E_y} + \nu_{yx}E_x + 2G_{xy}\left(1-\nu_{xy}\nu_{yx}\right)\right]$$

   e. For each stress value of interest calculate (N_x/b) by

$$\frac{N_x}{b} = \frac{9\sqrt{2}\,(1-\nu_{xy}\nu_{yx})^{1/2}\,\sigma_x^2}{\pi^2 K^{1/2}\,E_o^{1/2}\,(E_x E_y)^{1/4}}$$

   f. Determine (h_c/b) and (d_f/b) by

$$\frac{h_c}{b} = \frac{d_f}{b} = \left[\frac{6\,\sigma_x}{\pi^2 K\,(E_x E_y)^{1/2}}\right]^{1/2}$$

   g. Determine (t_f/b) and $t_c/b)$ by

$$\frac{t_f}{b} = \frac{t_c}{b} = \frac{3\sqrt{2}\,(1-\nu_{xy}\nu_{yx})^{1/2}\,\sigma_x}{\pi^2 K^{1/2}\,(E_x E_y)^{1/4}\,E_o^{1/2}}$$

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h. Determine the minimum weight by

$$\frac{W-W_{core}}{b} = \left(\frac{N_x}{b}\right)\frac{c}{\sigma_f}$$

3. **Panels in Which the Faces and Core are of Different Isotropic Materials**

The following procedures are applicable for stresses below the proportional limit of both face and core material.

a. Known quantities:

$$\theta = 0^\circ, \quad \nabla = \infty \quad , \quad a/b.$$

b. Select materials and face stress values of interest.

c. Calculate

$$\frac{E_c \bar{I}_c}{E_f \bar{I}_f} = \frac{1}{6}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_c}{E_f}\right) \quad \text{and} \quad R = \frac{1}{6}\left(\frac{\rho_c}{\rho_f}\right)\left(\frac{E_f}{E_c}\right)$$

d. For this value of $E_c \bar{I}_c / E_f \bar{I}_f$ use Figure 2 or 4 of NACA TN 2679 to determine K from the ordinate for the given value of a/b, or use Equation 6 of NACA TN 2679 to obtain the value.

e. Determine $(N_{x/b})$ for each σ_f by

$$\frac{N_x}{b} = \frac{\sqrt{6}\left(1-\nu_f^2\right)^{1/4}\left(1-\nu_c^2\right)^{1/4}}{\pi^2 K^{1/2}} \frac{E_c^{1/2}}{E_f^{5/2}} \left(\frac{\rho_f}{\rho_c}\right) R^{3/2} \sigma_f^2$$

f. Determine $d_{f/b}$ for each σ_f of interest by

$$\frac{d_f}{b} = \frac{2^{1/2}}{\pi K^{1/2}} \left(\frac{1-\nu_c^2}{1-\nu_f^2}\right)^{1/4} \frac{E_c^{1/2}}{E_f} R^{1/2} \sigma_f^{1/2}$$

g. Determine $h_{c/b}$ for each σ_f by

$$\frac{h_c}{b} = \left(\frac{\rho_f}{\rho_c}\right)^{1/2} \left(\frac{1-\nu_f^2}{1-\nu_c^2}\right)^{1/4} \frac{d_f}{b}$$

h. Determine $(t_{f/b})$ for each σ_f by

$$\frac{t_f}{b} = \frac{6^{1/2}}{\pi^2 K^{1/2}} \left(1-\nu_f^2\right)^{1/4}\left(1-\nu_c^2\right)^{1/4} \frac{E_c^{1/2}}{E_f^{1/2}} R^{1/2} \sigma$$

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i. Determine t_c/b for each σ_f by

$$\frac{t_c}{b} = \left(\frac{\rho_f}{\rho_c}\right)^{1/2}\left(\frac{1-\nu_c^2}{1-\nu_f^2}\right)^{1/4}\left(\frac{t_f}{b}\right)$$

j. Determine $(W - W_{ad})/b$ for each σ_f by

$$\frac{W - W_{ad}}{b} = 3\rho_f\frac{t_f}{b}$$

k. Determine σ_c for each σ_f to insure that both stresses are below the proportional limit, by

$$\sigma_c = \sigma_f\frac{E_c}{E_f}$$

4. **Panels in Which Faces and Core are of Different Ortho-tropic Materials**

The following procedures are applicable to stresses below the proportional limit of both face and core materials.

   a. Known quantities:

   $$\theta = 0^\circ, \quad \nabla = \infty, \quad a/b$$

   b. Select materials and face stress values of interest.

   c. Calculate

   $$\frac{E_c\bar{I}_c}{E_f\bar{I}_f} = \frac{1}{6}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}E_{cy}}{E_{fx}E_{fy}}\right)^{1/2} \text{ and } R = 1 + 2\left(\frac{E_c}{\rho_f}\right)\left(\frac{E_{cy}}{E_{cx}}\right)$$

   d. Calculate E_{o_i} $(i = c, f)$

   $$E_{oi} = \frac{1}{2}\left[\sqrt{E_{ix}E_{iy}} + \nu_{yxi}E_{ix} + 2G_{xyi}\left(1-\nu_{xyi}\nu_{yxi}\right)\right]$$

   e. For this value of $E_o\bar{I}_c/E_f\bar{I}_f$ use Figure 2 or 4 of NACA TN 2679 to determine the buckling coefficient K from the ordinate for the given value of a/b, or use equation 6 of NACA TN 2679 to obtain the value.

f. Determine $N_{x/b}$ for each σ_f by

$$\frac{N_x}{b} = \frac{\sqrt{6}\left(1-\nu_{xyf}\,\nu_{yxf}\right)^{1/4}\left(1-\nu_{xyc}\,\nu_{yxc}\right)^{1/4}}{\left(E_{0f}\,E_{0c}\right)^{1/4}\left(E_{fx}\,E_{fy}\right)^{1/4}\pi^2 K^{1/2}}\left(\frac{E_{cx}}{E_{fx}}\right)^{3/4}\left(\frac{\rho_f}{\rho_c}\right)R^{3/2}\sigma_f^2$$

g. Determine $\dfrac{h_c}{b}$ for each value of σ_f by

$$\frac{h_c}{b} = \left\{\frac{2}{\pi^2 K}\left(\frac{\rho_f}{\rho_c}\right)\left(\frac{E_{cx}}{E_{fx}}\right)\frac{R\,\sigma_f}{\left(E_{fx}\,E_{fy}\right)^{1/2}}\right\}^{1/2}$$

h. Determine $d_{f/b}$ for each value of σ_f by

$$\frac{d_f}{b} = \frac{h_c}{b}\left[\left(\frac{\rho_c}{\rho_f}\right)^2\left(\frac{1-\nu_{xyc}\,\nu_{yxc}}{1-\nu_{xyf}\,\nu_{yxf}}\right)\left(\frac{E_{0f}}{E_{0c}}\right)\left(\frac{E_{cx}}{E_{fx}}\right)\right]^{1/4}$$

i. Determine $(t_{c/b})$ for each value of σ_f by

$$\frac{t_c}{b} = \frac{\sqrt{6}}{\pi^2 K^{1/2}}\frac{\left(1-\nu_{xyc}\,\nu_{yxc}\right)^{1/2}}{E_{0c}^{1/2}\left[E_{fx}\,E_{fy}\right]^{1/4}}\left(\frac{\rho_f}{\rho_c}\right)^{1/2}\left(\frac{E_{cx}}{E_{fx}}\right)R^{1/2}\sigma_f$$

j. Determine $t_{f/b}$ for each value of σ_f by

$$\frac{t_f}{b} = \frac{\rho_c}{\rho_f}\left(\frac{t_c}{b}\right)\left[\frac{h_c/b}{d_f/b}\right]$$

k. Determine the minimum weight $(W - W_{ad})/b$ for each value of σ_f by

$$\frac{W-W_{ad}}{b} = 3\rho_f\left(\frac{t_f}{b}\right)$$

l. Determine σ_c for each σ_f to insure that both stresses are below the proportional limit

$$\sigma_c = \sigma_f\,\frac{E_{cx}}{E_{fx}}$$

CHAPTER 3.

METHODS OF STRUCTURAL OPTIMIZATION FOR FLAT, HAT-SHAPED CORRUGATED CORE SANDWICH PANELS SUBJECTED TO UNIAXIAL COMPRESSION

A. Introduction

Consider a flat corrugated core sandwich panel involving the following construction, shown in Figure 5.
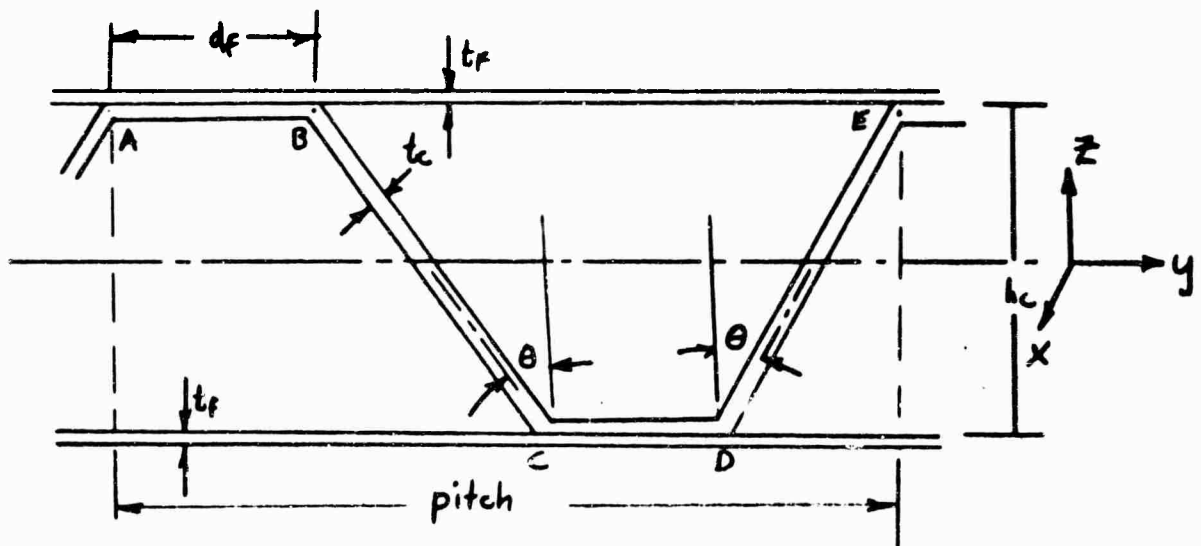

Figure 5

Hat-Shaped Corrugated Core Construction

In the geometry shown in Figure 5, there are five geometric parameters with which to optimize; namely, the core depth (h_c), the web thickness (t_c), the face thickness (t_f), the web angle (θ), and the length over which the core material is bonded or otherwise intimately joined with the

face material, d_f. This last variable does not exist in the triangulated core construction of Chapter 1.

The overall panel to be considered is shown in Figure 2.

The panel is considered to have failed if any of the following instabilities occur: overall instability, local face buckling in the region A to B, local face buckling in the region B to E, and web buckling.

Hence there are four modes of instability and five geometric parameters. To describe each mode of instability the analytical expressions used in the optimization study are considered the best available at this time.

B. **Elastic and Geometric Constants Associated with Hat-Shaped Corrugated Core Construction**

The elastic and geometric constants for this type of construction are given or can be derived from those given by Libove and Hubka in Reference 1. In the following, the overall geometry of Reference 1 has been simplified to the minor extent that the radii of curvature at points A, B, C, and D in Figure 5 have been taken as zero. This greatly simplifies the expression and does not alter the results appreciably.

The area of the core per unit width of corrugation crosssection parallel to the yz plane, $\overline{A}_c$ is given by

$$\overline{A}_c = \frac{t_c(h_c + d_f \cos\theta)}{\cos\theta \ (d_f + h_c \tan\theta)} \qquad \text{(in.)} \tag{3.1}$$

The moment of inertia of the core, per unit width of corrugation crosssection parallel to the yz plane, taken about the centroidal axis of the corrugation crosssection can be written as

$$\overline{I}_c = \frac{t_c h_c^{\ 2} (h_c + 3d_f \cos\theta)}{12 \cos\theta \ (d_f + h_c \tan\theta)} \qquad \text{(in.}^3) \tag{3.2}$$

The extensional stiffness of the plate in the x direction, EA_x is given by

$$\overline{EA}_x = E_c \overline{A}_c + 2 E_f t_f = \frac{E_c t_c(h_c + d_f \cos\theta)}{\cos\theta \ (d_f + h_c \tan\theta)} + 2 E_f t_f \qquad \text{(lbs./in.)} \tag{3.3}$$

The transverse shear stiffness in planes parallel

to the corrugation, the x direction, is found to be

$$D_{q_x} = \frac{G_c t_c h_c^2 \cos\theta}{(h_c + d_f \cos\theta)(d_f + h_c \tan\theta)} \quad (\text{lb./in.}) \qquad (3.4)$$

The transverse shear stiffness in planes perpendicular to the direction of the corrugations is given by

$$D_{q_y} = \frac{S h_c E_c}{(1-v_c^2)} \left(\frac{t_c}{h_c}\right)^3 \quad (\text{lbs./in.}) \qquad (3.5)$$

The values of S to use in this expression are given in Reference 1 by Figure 3, as functions of h_c/t_c, t_c/t_1, θ, and p/h_c. In this work, $p = d_f + h_c \tan\theta$, θ of Reference 3 is $(90^0 - \theta)$ in this report, and t_1 of Reference 3 is t_f in this report.

The moment of inertia per unit width, $\overline{I}_f$, of the faces considered as membranes with respect to the sandwich plate middle surface is found to be

$$\overline{I}_f = \frac{t_f h_c^2}{2} \qquad (3.6)$$

Since $t_f \ll h_c$ the core depth (h_c) can be taken as the distance between the centerlines of the faces.

C. <u>Governing Equations for Panels Composed of Isotropic Materials</u>

1. <u>Overall Instability</u>

The best expression describing the overall instability of a corrugated core sandwich panel under uniaxial compressive loads is given by Seide in Reference 3. It can be written as:

$$N_x = \frac{\pi^2 E_f \bar{I}_f K}{b^2} = \frac{\pi^2 E_f t_f h_c^2 K}{2b^2} \tag{3.7}$$

This is the same equation as (1.9). The buckling coefficient K is found from Reference 3, Equation 6. The equation for K is used instead of utilizing Figure 2 of Reference 3, because in numerical computations, the minimum weight configuration results in values which are difficult to interpolate using the Figure. Making use of the fact that, as stated by Seide, only one half sine wave will occur in the y direction the value of K can be given as

$$K = \frac{\left\{ \begin{array}{c} \left(\frac{m}{\beta} + \frac{\beta}{m}\right)^2 + (1-\nu_f^2)\eta\,\frac{m^2}{\rho^2} + \left[\frac{1}{2(1+\nu_f)}\left(\frac{m}{\beta} + \frac{\beta}{m}\right)^2\right. \\ \left. + \eta\left(1 + \frac{1-\nu_f}{2}\frac{m^2}{\beta^2}\right)\right]\left(r_x + \frac{m^2}{\beta}r_y\right) \end{array} \right\}}{\left\{ \begin{array}{c} 1-\nu_f^2 + \left[\frac{1-\nu_f}{2} + \frac{m^2}{\beta^2} + (1-\nu_f^2)\eta\,\frac{m^2}{\rho^2}\right]r_x + \left(1 + \frac{1-\nu_f}{2}\frac{m^2}{\rho^2}\right)r_y \\ + \left[\frac{1}{2(1+\nu_f)}\left(\frac{m^2}{\rho^2} + 1\right)^2 + \eta\,\frac{m^2}{\beta^2}\left(1 + \frac{1-\nu_f}{2}\frac{m^2}{\rho^2}\right)\right]r_x\,r_y \end{array} \right\}} \tag{3.8}$$

where m =   the number of half sine waves in the x direction

$$\beta = a/b$$

$$\eta = E_c \overline{I}_c / E_f \overline{I}_f$$

$$r_x = \frac{^2 E_f \overline{I}_f}{b^2 D_{q_x}}$$

$$r_y = \frac{^2 E_f \overline{I}_f}{b^2 D_{q_y}}$$

The general characteristics of this equation are discussed in more detail in Chapter 1, Section C-1.

2. Face Plate Instability

Referring to Figure 5, the facings can buckle in the regions between points B and E. Since the support conditions at B and E, the unloaded edges, are unknown precisely, it is conservative to assume that they are simply supported. For such a case the lower bound of the buckling coefficient for this condition, where the length to width ratio is greater than unity, is equal to 4. The well known buckling equation to describe this instability, written in terms of this construction, is found to be

$$\sigma_F = \frac{\pi^2 E_F t_F^2}{3(1-\nu_F^2)\left(d_F + 2h_c \tan\theta\right)^2} \tag{3.9}$$

Likewise, it is possible for the face plate between A and B to buckle. In this region, since the core material and the face material are intimately joined or bonded in some way,

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the plate element has a thickness of $(t_c + t_f)$. If the construction involves the same material for both facing and core, the expression for the critical stress is found to be, assuming simply supported edges,

$$\sigma_F' = \frac{\pi^2}{3(1-v^2)} \frac{E\left(t_c+t_F\right)^2}{d_f^2} \tag{3.10}$$

The expression will be more complicated if the face and core materials differ because there will be a shift in the neutral axis from the centroid.

3. Web Plate Instability

Similarly the local plate elements of the core case become unstable due to the core being directly subjected to a portion of the axial loading, N_x. The conservative assumption is also made here that the web elements from B to C and D to E in Figure 5 are simply supported along the unloaded edges, since the actual boundary conditions are somewhere between the simply supported and clamped boundary condition. Hence K = 4 for these elements which have a length in the x direction greater than the dimensions of B to C and D to E in Figure 5.

In terms of the symbols of Figure 5, the plate buckling equation is easily determined to be

$$\sigma_c = \frac{\pi^2 E_c}{3(1-v_c^2)} \frac{t_c^2}{h_c^2} \cos^2\theta \tag{3.11}$$

Equation (3.11) is of course identical to Equation (1.12).

4. Load-Stress Relationship

Following the developments of Chapter 1, Section C-4, it is easily shown that for uniform strain on the loaded edges

$$c_c = \sigma_F \frac{E_c}{E_f} \tag{3.12}$$

Likewise, it is seen that

$$N_x = \sigma_c \bar{A}_c + 2\sigma_F t_F \tag{3.13}$$

Substituting (3.1) and (3.12) into (3.13), the load-stress relationship for this construction is given by

$$N_x = \sigma_F \left[\frac{E_c}{E_f} \frac{t_c (h_c + d_F \cos\theta)}{\cos\theta (d_F + h_c \tan\theta)} + 2t_F \right] \tag{3.14}$$

Again relations (3.12) and (3.13) are only applicable when both the face material and core material are stressed below their proportional limit. However, when the core and face materials are the same then $\sigma_c = \sigma_f$ and Equation (3.14) applies over the entire stress range.

5. Weight Relation

The weight equation is seen to be

$$W = \rho_c \bar{A}_c + 2\rho_F t_F + W_{ad}$$

which for this particular geometry becomes

$$W - W_{ad} = \frac{\rho_c t_c (h_c + d_F \cos\theta)}{\cos\theta (d_F + h_c \tan\theta)} + 2\rho_F t_F \tag{3.15}$$

D. Structural Optimization of Panels with Faces and Core of the Same Isotropic Material

The governing equations pertaining to this construction given in the previous subsection can be simplified if the facing and core are made of the same isotropic materials; namely, $E_c = E_f = E$, $\nu_c = \nu_f = \nu$, and $\sigma_c = \sigma_f = \sigma$. The results are:

$$N_x = \frac{\pi^2 \bar{E}\, t_f h_c^2 K}{2b^2} \tag{3.16}$$

$$\sigma = \frac{\pi^2 \bar{E}\, t_f^2}{3(1-\nu^2)(d_f + 2h_c \tan\theta)^2} \tag{3.17}$$

$$\sigma = \frac{\pi^2 \bar{E}\,(t_c + t_f)^2}{3(1-\nu^2)\, d_f^2} \tag{3.18}$$

$$\sigma = \frac{\pi^2 \bar{E}}{3(1-\nu^2)}\, \frac{t_c}{h_c^2}\, \cos^2\theta \tag{3.19}$$

$$N_x = \sigma\left[\frac{t_c (h_c + d_f \cos\theta)}{\cos\theta\,(d_f + h_c \tan\theta)} + 2t_f\right] \tag{3.20}$$

$$W - W_{ad} = \rho\left[\frac{t_c (h_c + d_f \cos\theta)}{\cos\theta\,(d_f + h_c \tan\theta)} + 2t_f\right] \tag{3.21}$$

In the above, the $\bar{E}$ is a reduced modulus of elasticity if the stresses are above the proportional limit.

Immediately from (3.20) and (3.21), it is seen that

$$\frac{N_x}{b} = \left(\frac{W - W_{ad}}{b}\right)\frac{\sigma}{\rho} \quad\text{and}\quad \frac{W - W_{ad}}{b} = \left(\frac{N_x}{b}\right)\frac{\rho}{\sigma} \tag{3.22}$$

The philosophy of optimization expressed in Chapter 1, Section D is utilized here.

Looking at the set of Equations (3.16) through (3.22) the known or specified quantities are the applied load per inch (N_x) and the panel width (b) which can be grouped together as the load index (N_x/b), and the material properties E, ν and ρ. The buckling coefficient K is a slowly verying parameter of the dependent variables which is considered here as a constant, but will be discussed in the next section.

The dependent variables are t_f, h_c, t_c, θ, σ, d_f, and $W - W_{ad}$. Thus there are six equations and seven unknowns.

Turning first to Equations (3.17), (3.18), and (3.19), it is found that the minimum weight structure for the construction shown by Figure 5 occurs when

$$\theta = -30°$$

$$\text{and} \quad t_c = \frac{2 t_f h_c}{(\beta d_f - 2 h_c)} = \frac{2(1-\nu^2)^{\frac{1}{2}} h_c \sigma^{\frac{1}{2}}}{\pi E^{\frac{1}{2}}}$$

$$\text{and} \quad t_f = \frac{(\beta d_f - 2 h_c)(1-\nu^2)^{\frac{1}{2}} \sigma^{\frac{1}{2}}}{\pi E^{\frac{1}{2}}}$$

$$\left.\right\} \quad (3.23)$$

Thus the minimum weight panel with this type of construction has the geometry shown in the centerline sketch below.
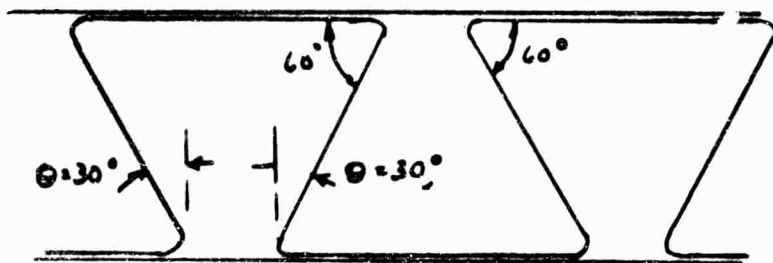
This leaves much to be desired from a practical viewpoint, because of the larger angles through which the core material must employ small radius bends in conjunction with the comparatively large regions AB over which loading is required. However, to have the angle $\theta > -30°$, implies that the critical stress for the plate element from A to B will be greater than either the critical stress for the plate element from B to E, or the core plate element from B to C.

Although a geometry in which $\theta > -30°$ is not a true minimum weight construction for hat-shaped corrugated core construction, it is of interest to investigate the "optimum" geometry under the constraint that $\theta \geqslant 0°$ (to avoid a small radius bend in the core material of greater than 90), which implies the elimination of Equation (3.18) in the following development. Thus, such construction can be compared to the triangulated core (truss core) and the web core construction, optimized in Chapters 1 and 2, respectively. Now, there are five equations and seven unknowns. It would therefore be desirable through manipulating the governing equations to obtain the weight equation in terms of two dependent variables. However, due to the complexity of the equations, it is not possible to uncouple the dependent variables in such a manner to accomplish this. However, appealing to the fact that the dimension d_f is often determined by manufacturing limitations and restrictions, or could even be specified by some other consideration, we can consider it as a constant, and investigate the optimum construction for a number of specific values of d_f. The weight equation

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can now be obtained in terms of the variables h_c and θ (although other variables could possibly have been employed).

$$W-W_{ad} = \frac{3\rho\left(d_f + 2h_c \tan\theta\right)^2 \pi^2 \bar{E} h_c^4 K^2 (1-\nu^2)}{4b^4 N_x} \qquad (3.24)$$

In the range of variable θ, $0° \leqslant \theta \leqslant 90°$, it is obvious that the weight will be least when $\theta = 0°$.

With $\theta = 0$, the set of governing equations becomes the following:

$$N_x = \frac{\pi^2 \bar{E} t_f h_c^2 K}{2b^2} \qquad (3.25)$$

$$\sigma = \frac{\pi^2 \bar{E} t_f^2}{3(1-\nu^4) d_f^2} \qquad (3.26)$$

$$\sigma = \frac{\pi^2 \bar{E} t_c^2}{3(1-\nu^4) h_c^2} \qquad (3.27)$$

$$N_x = \sigma\left[\frac{t_c(h_c + d_f)}{d_f} + 2t_f\right] = (W-W_{ad})\frac{\sigma}{\rho} \qquad (3.28)$$

$$W-W_{ad} = \rho\left[\frac{t_c(h_c + d_f)}{d_f} + 2t_f\right] = N_x \frac{\rho}{\sigma} \qquad (3.29)$$

With the set (3.25) through (3.29), it is still not possible to separate the variables in such manner as to obtain explicit solutions, as was done in the case of the triangulated core construction, the web-core construction, or the honeycomb core construction in Reference 6. A numerical, iterative scheme must therefore be employed.

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A great variety of iterative techniques could be employed. However, experience with numerical examples, discussed in Chapter 4 of this report, indicates the following as the most direct.

From (3.26) it is easily seen that

$$\frac{t_E}{b} = \frac{3^{1/2} (1-\nu)^{1/2} \left(\frac{d_E}{b}\right) \sigma^{1/2}}{\pi \bar{E}^{1/2}}$$

(3.30)

From (3.25) and (3.30),

$$\frac{h_s}{b} = \frac{2^{1/2} (N_x/b)^{1/2}}{\pi^{1/2} \bar{E}^{1/4} K^{1/2} 3^{1/4} (1-\nu^2)^{1/4} \left(\frac{d_E}{b}\right)^{1/2} \sigma^{1/4}}$$

(3.31)

From (3.27) and (3.31), it is seen that

$$\frac{t_s}{b} = \frac{(3)^{1/4} (1-\nu)^{1/2} (2)^{1/2} \left(\frac{N_x}{b}\right)^{1/2} \sigma^{1/4}}{\pi^{3/2} \bar{E}^{3/4} K^{1/2} \left(\frac{d_E}{b}\right)^{1/2}}$$

(3.32)

Substituting (3.30), (3.31), and (3.32) into (3.28), the following relation is obtained.

$$\frac{N_x}{b} = \sigma \left\{ \frac{2(3)^{1/2}(1-\nu)^{1/2}\left(\frac{d_E}{b}\right)\sigma^{1/2}}{\pi \bar{E}^{1/2}} + \frac{(3)^{1/4}(2)^{1/2}(1-\nu)^{1/4}(N_x/b)^{1/2}\sigma^{1/4}}{\pi^{3/2} \bar{E}^{3/4} K^{1/2} (d_E/b)^{1/2}} \right. $$
$$\left. + \frac{2(N_x/b)}{\pi^2 \bar{E} K (d_E/b)^2} \right\}$$

(3.33)

This is the "universal relation" relating the applied load ($N_{x/b}$) to the stress σ, for a given or specified value of (d_E/b).

In this optimization procedure, use is made of the fact that in solving (3.33) for a given load $(N_{x/b})$, for various specified values of $(d_{f/b})$, that value of $(d_{f/b})$ which results in the highest value of stress σ is the value of $(d_{f/b})$ which will result in minimum weight, according to Equation (3.22), since $\dfrac{W - W_{ad}}{b}$ is inversely proportional to σ.

The iterative procedure is now explained in detail, for a panel with specified load index $(N_{x/b})$, to obtain a minimum weight construction.

1. Known quantity is $\sigma = 0$.

2. Select a material of construction.

3. Estimate a value of the buckling coefficient K

4. Select a value of $(d_{f/b})$

5. Estimate a stress value

6. Determine $\overline{E}$ for the assumed stress in Step 5. Below the proportional limit E will be used, above the proportional limit $\overline{E} = \sqrt{EE_t}$ may be used (see footnote, Chapter 1, Section H)

7. Substitute these into Equation (3.33) to see if an equality is obtained. If the right hand side is less than $N_{x/b}$ specified, the assumed stress is too low, and conversely. Repeat steps 5,6, and 7 until an equality is obtained. This is the solution for the selected value of $(d_{f/b})$.

8. Select another value of $(d_{f/b})$ and repeat steps 4 through 7 until a value of $(d_{f/b})$ is found which

has the highest value of σ as a solution.

9. For the value of $(d_{f/b})$ which has the highest value of σ as its solution, solve for $(t_{f/b})$, $(t_{c/b})$, and $(h_{c/b})$ using Equations (2.27), (2.28), and (2.29).

10. With these values, using relations derived previously, solve for

a. $\eta = \dfrac{\bar{I}_c}{\bar{I}_F} = \dfrac{1}{6} \dfrac{(t_{c/b})}{(t_{F/b})} \dfrac{\left[\dfrac{h_c}{b} + 3\left(\dfrac{d_F}{b}\right)\right]}{\left(\dfrac{d_F}{b}\right)}$

b. $r_y = \dfrac{\pi^2 E \bar{I}_F}{b^2 D_{\phi y}} = \dfrac{\pi^2}{2} \left(\dfrac{t_F}{b}\right)\left(\dfrac{h_c}{b}\right) \dfrac{1-\nu^2}{S} \dfrac{(h_{c/b})^3}{(t_{c/b})^3}$

   where S is found in Figure 3, Reference 1

   Note: θ of Reference 3 is $(90^\circ - \theta)$ in this report and $p = d_f$.

c. $r_x = \dfrac{\pi^2 (1+\nu)(t_{F/b})\left[(h_{c/b}) + (d_{F/b})\right](d_{F/b})}{(t_{c/b})}$

11. Solve Equation (3.8) for K. Because of the characteristics of K, one may assume a value of $m = 1$ and $\beta = 1.6$ for a good approximation. However if an exact value of K is required one can let $m = 1$, and vary β until a minimum is obtained.

12. Compare the calculated value of K from step 11 with the value assumed in step 3. If they do not match, repeat steps 4 through 12 until the assumed value equals the calculated value.

13. For the final solution of step 12, values of $t_{f/b}$, $t_{c/b}$, and $h_{c/b}$ will have been calculated in step 9 and

the optimum stress from step 8, now calculate the weight by

$$\frac{W - W_{ad}}{b} = (\frac{N_x}{b}) \frac{\rho}{\sigma}$$

The optimization will now be complete.

The optimization of this type construction involving different core and face materials, or orthotropic materials would be increasingly complex.

CHAPTER 4

EXAMPLES OF OPTIMIZATION STUDIES OF FLAT CORRUGATED CORE SANDWICH PANELS SUBJECTED TO UNIAXIAL IN-PLANE COMPRESSION

As an example of the design procedures given in Chapter 1, Section E of this report for triangulated core (single truss core) panels, design curves have been constructed for the following material systems, where the numbers will be used to identify the materials in the Figures which follow:
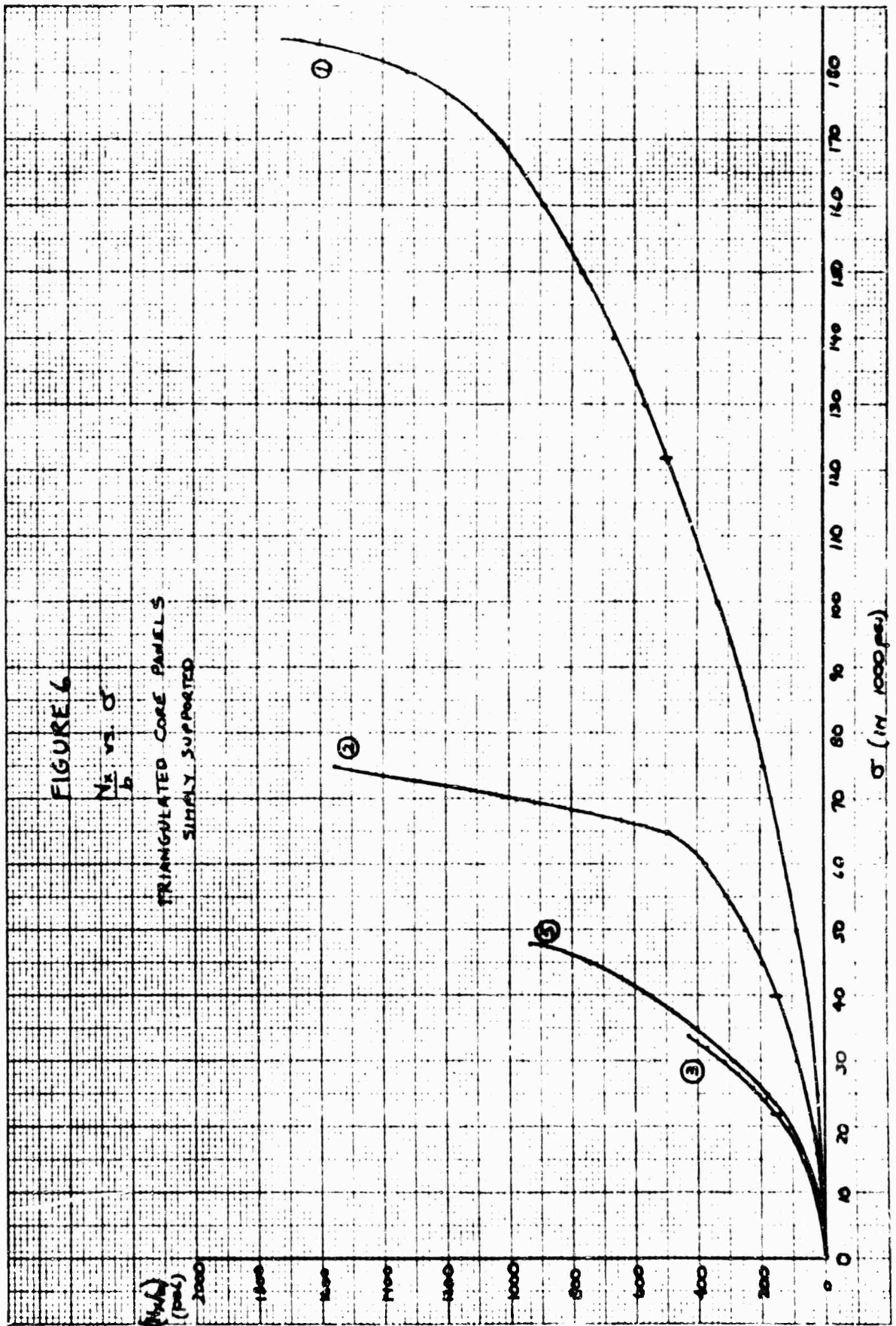
1. AISI 4340 steel: 200,000 psi level. Source - MIL-HDBK 5, page 2.3.1.2.4.(a).

2. 7075-T6 aluminum (clad). Source - MIL-HDBK 5, page 3.2.7.1.6.(a).

3. 181 glass fabric laminate with polyester resin. Source - ANC-17, page 33

4a. Faces - AISI 4340 steel: 200,000 psi level
    Core - 7075-T6 aluminum (clad)
    Sources - see materials 1 and 2 above

4b. Faces - 7075-T6 aluminum (clad)
    Core - AISI 4340 steel: 200,000 psi level
    Sources - see materials 1 and 2 above

5. 8994 - 181 HTS glass fabric, ERSB - 0111 resin
   Source - letter from K. H. Boller, Forest Products Lab. to R. J. Molella, N.A.E.C., dated September 30, 1964

6. 143 glass fabric laminate with polyester resin
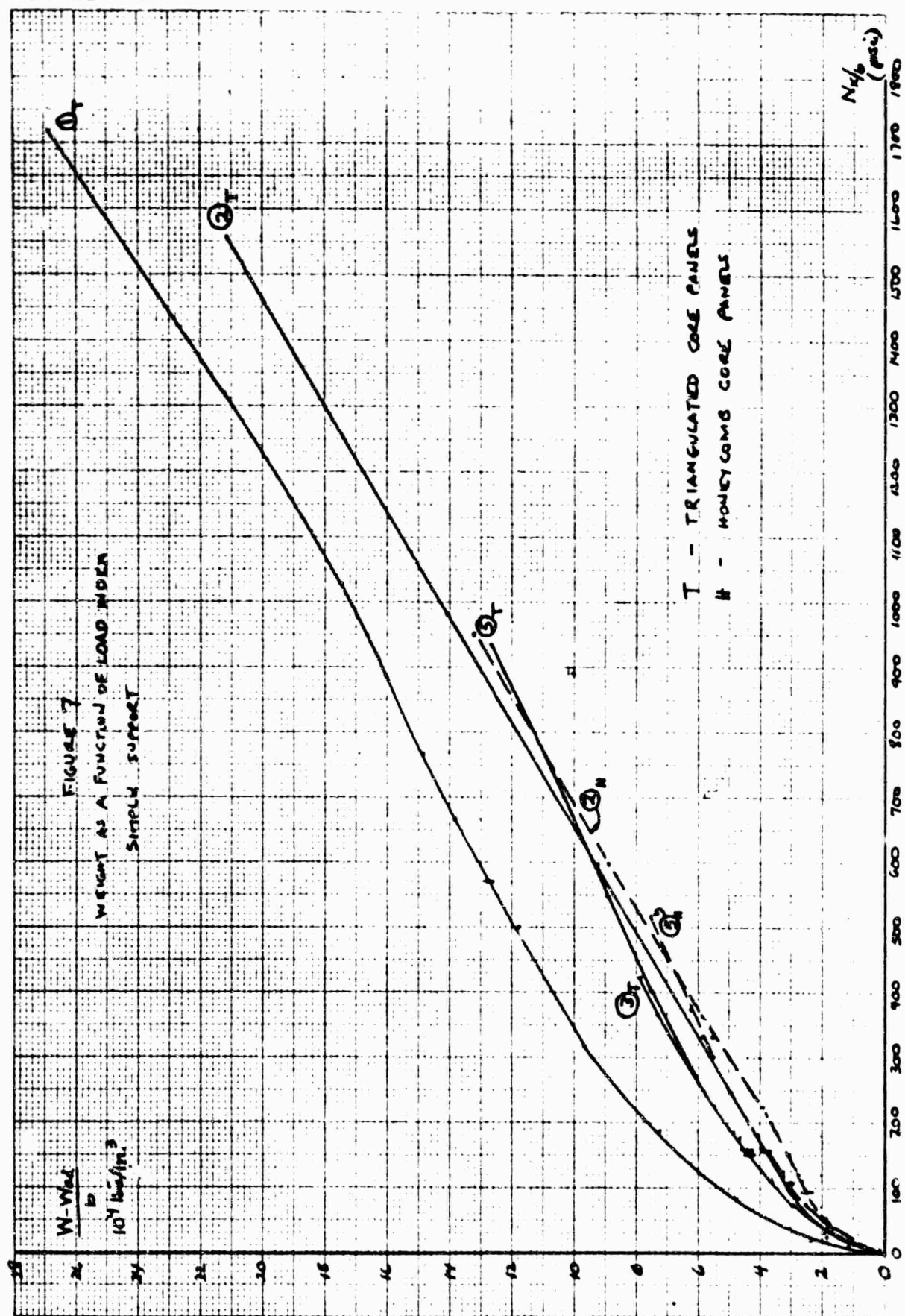   Source - MIL-HDBK 17 - CH - 1

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7. 143 glass fabric laminate with epoxy resin

   Source - MIL-HDBK 17 - CN - 1

8. 181 glass fabric laminate with epoxy resin

   Source - MIL-HDBK 17 - CN - 1, p. 2-54.

9. 120 glass fabric with epoxy resin

   Source - MIL-HDBK 17 - CN - 1

10. Cross Rolled Beryllium

   Source - "Beryllium in Aerospace Structures"

All calculations have been carried out for the panels with simply supported edges. Materials 1, 2, 3, 5, and 10 are isotropic. Materials 4a and 4b, although isotropic, involve cores and faces of different materials. Materials 6 through 9 are orthotropic.

Figure 6 provides the "universal" curves for the isotropic materials, where both core and face are the same material, where the stresses have been included to the limit to where tangent moduli curves extend. The tick marks indicate the proportional limit. As stated previously for each specified load this curve shows the unique value of stress which results in a minimum weight structure.

Figure 7 presents the weight of optimum (minimum weight structure for the materials shown in Figure 6. The subscript T indicates triangulated core construction. Dashed curves of material 2 and 5 have been included from Reference 8 for honeycomb sandwich construction. It is seen that the aluminum truss core construction weighs less than the steel

FIGURE 6

$\frac{N_x}{b}$ vs. σ

TRIANGULATED CORE PANELS
SIMPLY SUPPORTED

FIGURE 7

WEIGHT AS A FUNCTION OF LOAD UNDER
SIMPLY SUPPORT

T - TRIANGULATED CORE PANELS

H - HONEYCOMB CORE PANELS

truss core construction over the entire stress range that
aluminum is applicable, however, there is a range of very high
loading in which only the steel truss core construction can be
used. It is also seen that at low values of load index the
aluminum truss core construction weighs less than the reinforced
plastic truss core construction, but at higher load values the
converse is true.

It is also seen that the honeycomb construction in
all cases weighs less than the triangulated core (truss core)
construction over the range of loads that honeycomb core con-
struction can be used. However, depending upon the method of
joining faces to core the adhesive weight, W_{ad}, which must be
added in each case may be the deciding factor in whether the
total weight, W, is less for one or the other construction.

However, it is important to note that in each case the honey-
comb core construction is restricted by the ultimate compressive strength
of the face material in the amount of load the panel can carry. For loads
which exceed the ultimate strength of the faces of the honeycomb core panels,
the corrugated core construction is the only one which can be used for optimum
design.

Figure 8 presents the comparative weights for all
ten material systems listed previously, over the elastic range,
i.e. stress below the proportional limit. The Figure illustrates
several interesting points for room temperature truss core
panels under axial compressive loads. It is seen that
beryllium is significantly better than all other materials
investigated, but has the disadvantage that only low loads can
be carried and yet retain the beryllium in the elastic range

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FIGURE 8

WEIGHT AS A FUNCTION OF LOAD INDEX
TRIANGULATED CORE PANEL
SIMPLY SUPPORTED

NOTE: CURVES INCLUDE RANGE ONLY TO
PROPORTIONAL LIMIT OF MATERIAL
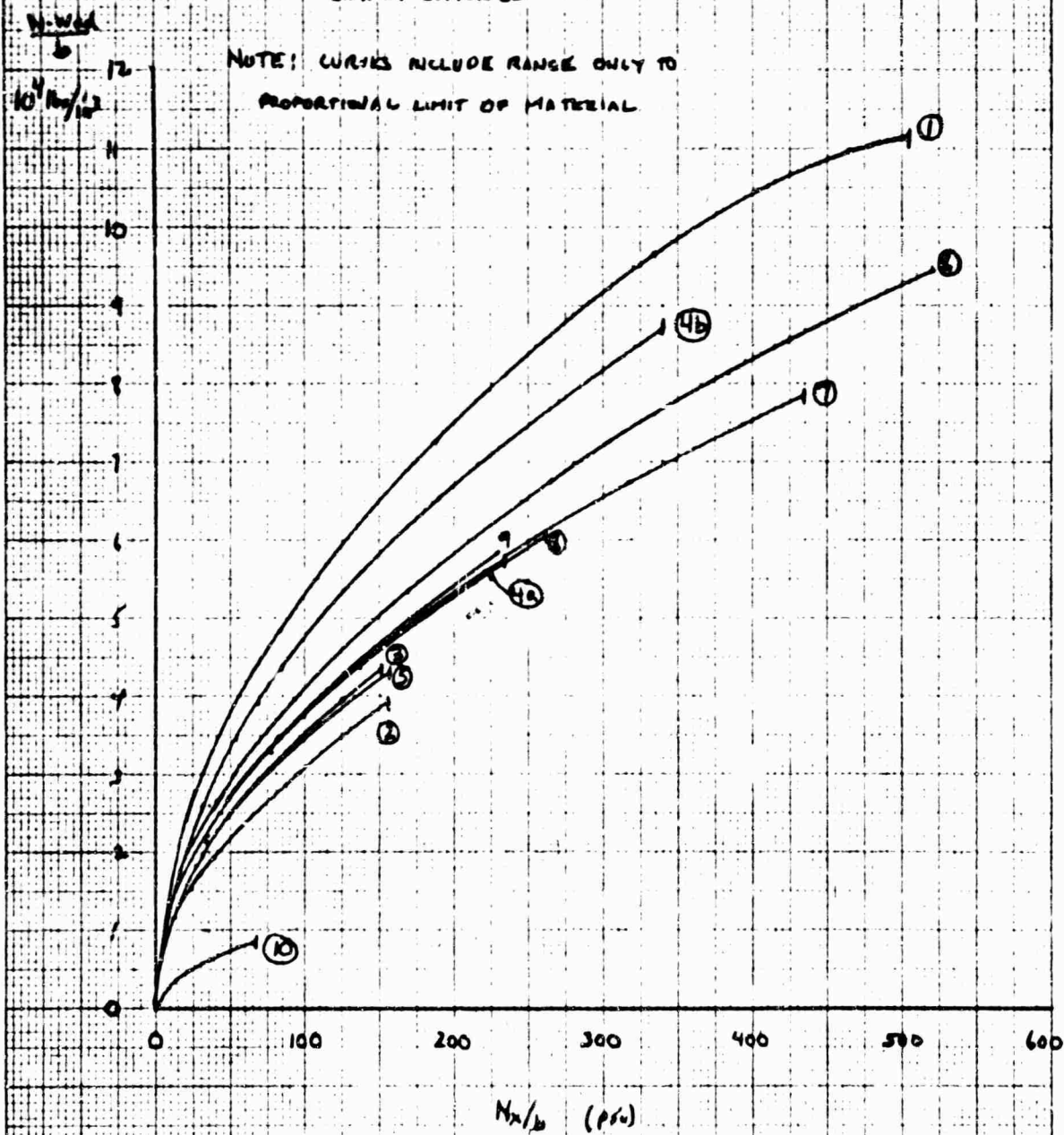
with optimum construction. However, it is significantly superior to other systems that a non-optimum configuration might weigh significantly less than an optimum construction of another material. Further, the non-optimum construction might increase the load carrying ability for stresses still in the elastic range. To reiterate, beryllium looks so promising that even a non-optimum construction which would extend curve 10 upward and to the right might still be superior to a minimum weight (optimum) construction of some other material.

It is seen that 7075-T6 aluminum is the second best of those investigated, over the range in which it ' licable. It is seen that aluminum is somewhat superior to t $\cancel{4}$-181 HTS glass fabric with ER8B - 0111 resin material over the entire range of applicability.

However, it is seen that the orthotropic reinforced plastics, materials 6, 7, 8, and 9, are all very close together over the lower load range, but the 143 glass fabric laminated with either polyester or epoxy resins remain in the elastic range to significantly higher loads than panels made of 7075-T6 aluminum for instance.

It is seen that steel construction is never competitive, and that for combinations of steel and aluminum, it is much better to use the steel for the faces and the aluminum for the core material.

In comparing the web-core construction of Chapter 2 with the triangulated core construction of Chapter 1, use is made

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of Equations (1.36) and (2.52), for a panel with face and core
of the same isotropic material. It is found that for any load
index, $(N_{x/b})$, and for stresses below the proportional limit,

$$\frac{\left(\frac{W - W_{ad}}{b}\right)_{web}}{\left(\frac{W - W_{ad}}{b}\right)_{Triang.}} = \frac{2^{3/4}}{5^{1/2}} \left(\frac{K_{triang}}{K_{web}}\right)^{1/4} \tag{4.1}$$

The buckling coefficient for the triangulated core for
a panel with simply supported edges, since $E_c\overline{I}_c/E_f\overline{I}_f$, and since
$\nabla \ll 1$ for the optimum configuration is 4.70 for a/b $\geqslant$ 1.2.

However, the buckling coefficient for the web core
panel, since $\nabla = \infty$ varies with a/b over the entire range.
Since $E_c\overline{I}_c/E_f\overline{I}_f = 1/6$ for the optimum construction the buckling
coefficient is found to be K = 1.0, 0.87, and 0.67 for a/b of
1.2, 1.6, and 2.0 respectively. Substituting these values into
Equation (4.1) the ratio is seen to be 1.11, 1.15, and 1.225
respectively for the a/b values calculated. Hence, optimum web
core construction weighs 11%, 15% and 22.5% more than optimum
truss core construction for a/b = 1.2, 1.6, and 2.0 respectively.
Physically, the reason why the web core construction is less
attractive is because of its lack of ability to resist transverse
shear in the y direction. It is true that as the length to width
ratio decreases below unity, the optimum web core construction
will weigh less than the optimum truss core construction.

Turning now to the comparison between the hat-shaped
corrugated core and the triangulated core (truss core) type

construction, an investigation was carried out with 7075 - T6 aluminum panels. For the truss core type construction for a face stress of 40,000 psi the $N_{x/b}$ is 156; for a face stress of 20,000 psi the $N_{x/b}$ is 39. Hence, the hat core construction was investigated for two cases, $N_{x/b} = 156$ and $N_{x/b} = 39$.

The optimum angle $\theta = 0^{\circ}$* is shown in Chapter 3. Following the design procedures given in Section D of that chapter the following is seen for optimum configurations.

Example 1 $N_{x/b} = 156$ psi, 7075 - T6 aluminum

Hat Core	Triangulated Core
$\sigma = 33,000$ psi	$\sigma = 40,000$ psi
$\theta = 0^{\circ}$	$\theta = 32.4^{\circ}$
$t_{f/b} = 1.33 \times 10^{-3}$	$t_{f/b} = 1.04 \times 10^{-3}$
$t_{c/b} = 10.9 \times 10^{-4}$	$t_{c/b} = 9.7 \times 10^{-4}$
$h_{c/b} = 0.038$	$h_{c/b} = 0.02526$
$(W-W_{ad/b}) = 4.72 \times 10^{-4}$ lb/in^3	$(W-W_{ad/b}) = 3.903 \times 10^{-4}$
$\dfrac{d_f}{b} = 0.045$	

* See discussion on page 77.

Example 2 $N_{x/b}$ - 39, 7075 - T6 aluminum

Hat Core	**Triangulated Core**
σ = 16,000 psi	σ = 20,000 psi
θ = 0°	θ = 32.4°
$t_{f/b}$ = 6.18 x 10^{-4}	$t_{f/b}$ = 5.2 x 10^{-4}
$t_{c/b}$ = 5.66 x 10^{-4}	$t_{c/b}$ = 4.867 x 10^{-4}
$h_{c/b}$ = 0.0281	$h_{c/b}$ = 0.0179
$(W-W_{ad/b})$ = 2.36 x 10^{-4} lb./in^3	$(W-W_{ad/b})$ = 1.951 x 10^{-4} lb/in^3

$\dfrac{d_f}{b}$ = 0.030

It is seen that in Example 1 the hat core construction is 20.5% heavier than the triangulated core construction subjected to the same load $(N_{x/b})$; in Example 2 it is 21% heavier compared on an equal loading basis. It is felt that in every case the triangulated core construction will be superior to the hat shaped core construction; hence, further examples (which are laborious) were not carried out, nor were optimization procedures derived for dissimilar materials in face and core, or for orthotropic materials.

CHAPTER 5

CONCLUSIONS

Utilizing the methods of structural optimization
derived in Chapters 1 and 2 of this report, and the methods
derived in Reference 6, optimum honeycomb sandwich construction
weighs less than optimum triangulated, corrugated core construc-
tion, which in turn weighs less than hat-shaped corrugated core
construction, over the load range where comparisons are possible.
However, the triangulated core construction can be used for loads
significantly greater than is possible with honeycomb sandwich
construction. Also optimum web core construction weighs more
than optimum triangulated core construction where the length -
width ratio of the panel equals to or exceeds unity for the
examples studied.

In the triangulated core construction (single truss
core), in which the faces and core are composed of the same
material, be it isotropic or orthotropic, it is found that the
web angle Θ (see Figure 1) is always constant ($\Theta = Sin^{-1}$
$(2/7)^{1/2} = 32.4^{\circ}$) for optimum, minimum weight construction, and
that the weight of the core is always $7/8$ the weight of the
faces. These are independent of the materials used, the
boundary conditions or the magnitude of the load index.

For the web core construction it is found that for
any material system, minimum weight occurs where $\Theta = 0^{\circ}$ and the
face weight is twice the core weight. In addition, when the

same material is used in both face and core, be it isotropic or orthotropic, the face thickness t_f equals the core web thickness t_c, and the core depth (h_c) equals the spacing between webs (d_f), in optimum construction.

One benefit derived by the development of methods of analysis for optimum (minimum weight design) structures, other than the obvious benefit, is that it enables the designer to compare the absolute minimum weight construction with the construction employing commercially available sizes that approximate the actual optimum dimensions. In this way he can more rationally assess the following considerations: the weight penalty of using commercially available materials or the cost penalty of using non-commercially available sizes to obtain minimum weight. Obviously, this is a function of the specific application.

It is also recommended that a test program be designed and executed to evaluate the present optimization procedures.

APPENDIX A

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13. ABSTRACT

In this report is presented the development of rational methods of
structural optimization for flat, corrugated core and web core sandwich
panels subjected to uniaxial compressive loads. These methods provide
a means by which such panels can be designed with minimum weight for a
given load index, panel width, panel length, and face and core materials.
Methods presented will permit a rational material selection by comparing
the weights of the optimum construction for several material systems as a
function of load index. The methods account for both isotropic or
orthotropic face and core materials and various boundary conditions.

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	ROLE	WT	ROLE	WT	ROLE	WT
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